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# Parameter bounds evaluation for linear system with output backlash ⋆

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**Abstract:** In this paper a procedure is presented for deriving parameters bounds of linear systems with output backlash when the output measurement errors are bounded. First, using steady-state input/output data, parameters of the backlash are bounded. Then, given the estimated uncertain backlash and the output measurements collected exciting the system with a PRBS, bounds on the unmeasurable inner signal are computed. Finally, such bounds, together with the input sequence, are used for bounding the parameters of the linear block.

## 1. INTRODUCTION

Nonlinearities in actuators and sensors commonly employed in control systems can introduce a variety of interesting although sometimes undesirable behaviours that, in general, may be responsible of phenomena such as delays and oscillations which, in turn, may cause inaccuracy and, more generally, limitations on both the static and the dynamic performance of control systems (see, e.g. Tao and Kokotovic [1996]). Backlash (see Fig. 1) is one of the most important nonlinearities that strongly affects the chosen control strategies in the industrial machines, for example, deteriorating the overall performances: its presence gives rise to inaccuracies in the position and velocity. This particular nonlinearity, which can be classified as a dynamic (i.e., with memory) and hard (i.e. non-differentiable) one, commonly occurs in mechanical, hydraulic and magnetic components, e.g.: bearings, gears and impact dampers (see, e.g. Nordin and Gutman [2002]). It can arise from unavoidable manufacturing tolerances or sometimes can be deliberately incorporated in the system in order to cope with thermal expansion Bapat et al. [1983]. The interested reader is referred to Tao and Kokotovic [1996] for real-life examples of systems with either input or output backlash nonlinearities. The control of systems with backlash has been investigated by several authors and a number of approaches can be found in the literature. For example, in order to cope with the limitations caused by the presence of backlash, either robust or adaptive control techniques can be successfully employed (see, e.g., Corradini et al. [2004], and Tao and Canudas de Wit [Eds.] respectively) which, on the other hand, require the characterization of the nonlinear dynamic block. Surprisingly, there are only few contributions in the literature on the identification of systems with nonstatic hard nonlinearities (Bai [2002], Cerone and Regruto [2007]). Therefore, the identification of systems with unknown backlash is an open theoretical problem of major relevance to applications.
The configuration we are dealing with in this paper, shown in Fig. 2, closely resembles that of a Wiener model which in turn consists of a linear dynamic system followed by a static nonlinear part $\mathcal{N}$. The identification of such a model relies solely on input-output measurements, while the inner signal $x_t$ is not assumed to be available. The identification of Wiener models has attracted the attention of many authors Billings [1980] exploiting a number of different techniques. The main difficulty in the identification of Wiener systems is that the internal signal is not available for measurement. However, under the assumption of invertible nonlinearities, which is a common one, the inner signal can be recovered from the output measurements through inversion of the previously estimated nonlinearity. Unfortunately, many output nonlinearities encountered in real world problems are non-invertible Wigren [1998], thus the invertibility assumption appears to be quite restrictive. Removal of such an hypothesis makes the consistent evaluation of the inner signal sequence a difficult task even in the case of exactly known nonlinearities.
It must be stressed that existing identification procedures mostly require that the nonlinearity be static and differentiable, usually a polynomial (see e.g., Bai [1998], Cerone and Regruto [2003], Narenda and Gallman [1966] and the references therein). On the side of linear systems with hard nonlinearities, Bai [2002] considers the case of nonlinearities parameterized by one parameter. The proposed algorithm, based on the idea of separable least squares, can be applied to several common static and nonstatic input nonlinearities.
In identification, a common assumption is that the measurement error $\eta_t$ is statistically described. A worthwhile alternative to the stochastic description of measurement errors is the bounded-errors characterization, where uncertainties are assumed to belong to a given set. In the bounding context, all parameter vectors belonging to the *Feasible Parameter Set* (FPS), i.e. parameters consistent with the measurements, the error bounds and the assumed model structure, are feasible solutions of the identification problem. The interested reader can find further details on this approach in a number of survey papers (see, e.g., Milanese and Vicino [1991], Walter and Piet-Lahanier [1990]).
In this paper we present a scheme for the identification of linear systems with output backlash. More precisely, we address the problem of bounding the parameters of a stable single-input single-output SISO discrete time linear

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system with unknown backlash at the output (see Fig. 2) when the output error is considered to be bounded. Note that the inner signal $x(t)$ is not supposed to be measurable. To the author's best knowledge, no contribution can be found in the literature which address the above described identification problem.
The paper is organized as follow. Section 2 is devoted to the formulation of the problem. In Section 3, parameters of the nonlinear block are tightly bounded using input-output data collected from the steady-state response of the system to a suitably filtered square wave input. Then, in Section 4, through a dynamic experiment, for all $u_t$ belonging to a Pseudo Random Binary Signal (PRBS) sequence $\{u_t\}$, we compute tight bounds on the inner signal, through a suitable inversion of the output, which, together with the corresponding input sequence are used for bounding the parameters of the linear part in Section 5. A simulated example is reported in Section 6.

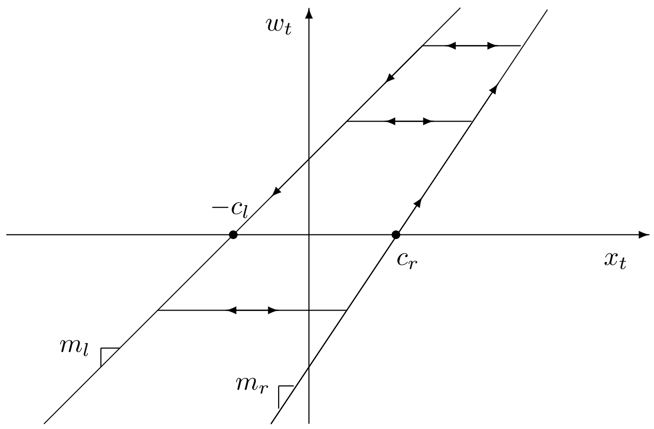


Fig. 1. Backlash.

## 2. PROBLEM FORMULATION

Consider the SISO discrete-time linear system with output backlash depicted in Fig. 2, where

$$x_t = G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} u_t. \quad (1)$$

$A(\cdot)$ and $B(\cdot)$ are polynomials in the backward shift operator $q^{-1}$, $(q^{-1}w_t = w_{t-1})$, $A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{na} q^{-na}$ and $B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{nb} q^{-nb}$. In line with the work done by a number of authors, in the contest of identification of block oriented systems, we assume that (i) the linear system is asymptotically stable (see, e.g., Stoica and Söderström [1982], Krzyżak [1993], Lang [1993, 1997], Sun et al. [1999]); (ii) $\sum_{j=0}^{nb} b_j \neq 0$, that is, the steady-state gain is not zero (see, e.g. Lang [1993, 1997], Sun et al. [1999]).The nonlinear block transforms $x_t$ into the noise-free output $w_t$ according to the following map (see, e.g., Tao and Kokotovic [1996])

$$w_t = \begin{cases} m_l(x_t + c_l) & \text{for } x_t \leq z_l \\ m_r(x_t - c_r) & \text{for } x_t \geq z_r \\ x_{t-1} & \text{for } z_l < x_t < z_r \end{cases} \quad (2)$$

where $m_l > 0$, $m_r > 0$, $c_l > 0$, $c_r > 0$ are constant parameters characterizing the backlash and

$$z_l = \frac{w_{t-1}}{m_l} - c_l, \qquad z_r = \frac{w_{t-1}}{m_r} + c_r \quad (3)$$

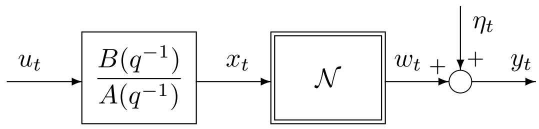


Fig. 2. Single-input single-output discrete-time linear system with output backlash $\mathcal{N}$.

are the $u$-axis values of intersections of the two lines, with slopes $m_l$ and $m_r$, with the horizontal inner segment containing $w_{t-1}$. The backlash characteristics is depicted in Figure 1.

Let $y_t$ be the noise-corrupted output

$$y_t = w_t + \eta_t. \quad (4)$$

where $\eta$ is known to range within given bounds $\Delta\eta_t$, i.e.,

$$| \eta_t | \leq \Delta\eta_t. \quad (5)$$

Unknown parameter vectors $\gamma \in R^4$ and $\theta \in R^p$ are defined, respectively, as

$$\gamma^T = [\ \gamma_1 \quad \gamma_2 \quad \gamma_3 \quad \gamma_4\ ] = [\ m_l \quad c_l \quad m_r \quad c_r\ ], \quad (6)$$

$$\theta^T = [\ a_1 \quad \ldots \quad a_{na} \quad b_0 \quad b_1 \quad \ldots b_{nb}\ ], \quad (7)$$

where $na + nb + 1 = p$. It is easy to show that the parameterization of the structure of Fig. 2 is not unique. To get a unique parameterization, in this work we assume, without loss of generality, that the steady-state gain of the linear part be one, that is

$$g = \frac{\sum_{j=0}^{nb} b_j}{1 + \sum_{i=1}^{na} a_i} = 1 \quad (8)$$

In this paper we address the problem of deriving bounds on parameters $\gamma$ and $\theta$ consistently with given measurements, error bounds and the assumed model structure.
First, exploiting $M$ steady-state input-output data, one gets the feasible parameter set $\mathcal{D}_\gamma$ of the nonlinear block parameters, which is a convex polytope. Then, given the estimated uncertain nonlinearity $\mathcal{N}(x_t, \gamma)$ and the output measurements collected exciting the system with a PRBS, bounds on the inner signal $x_t$ are computed through a suitable inversion of the output sequence.
Last, the bounds computed in the second stage, together with the input PRBS, are used to obtain a polytopic outer approximation of the exact feasible parameter set of the linear system. The proof of all the propositions presented in the paper can be found in found in Cerone et al. [2009].

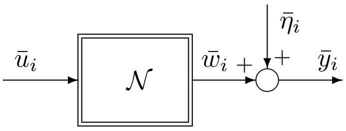


Fig. 3. Steady-state behaviour of the system under consideration when $g = 1$.

## 3. EVALUATION OF BOUNDS ON THE BACKLASH PARAMETERS

### 3.1 Input design for the steady-state experiment

In this section we exploit steady-state operating conditions to bound the parameters of the backlash. The output response of the linear block to a square wave input may show oscillations during the transients, which excite the hysteretic behaviour of the backlash, leading to possible constant output sequences in presence of non-constant input sequences. When such a case occurs, the relation between the input and the output of the backlash is no longer static and, as a consequence, the steady-state value of the backlash input is no longer uniquely related to the steady-state value of the input $u_t$ since it also depends on some past input/output samples. On the contrary, under assumption (8), the steady-state output of the backlash is uniquely related to the steady-state value of $u_t$ when $x_t$ does not overshoot its steady-state value. Thus, in order to estimate the backlash parameters, we look for an input signal $u_t$ such that the output of the linear dynamic block $x_t$ does not overshoot its steady-state value. A possible idea to design such an input is to obtain $u_t$ by filtering a square wave signal by means of a suitable linear filter. More specifically let us consider a linear low pass filter with transfer function $F(z) = (1-a)/(z-a)$. Applying a step signal $r_t$ of amplitude $\beta$ at the input of the filter and using the output of the filter as input of the linear block $G(q^{-1})$ we get:

$$X(z) = G(z)U(z), \quad U(z) = F(z)R(z) \tag{9}$$

where $U(z)$ and $X(z)$ are the z-transforms of the sequences $\{u_t\}$ and $\{x_t\}$ respectively, $G(z)$ is the transfer function of $G(q^{-1})$ and $R(z) = \beta/(z-1)$.
The design of the filter is based on the following result:

*Proposition 1.* For any constant $\varepsilon > 0$ there exists a value of the filter pole $a \in [0, 1[$ such that:

$$x_t = gu_t + e_t, \quad |e_t| \leq \varepsilon \ \forall t, \tag{10}$$

where $g$ is the steady-state gain of $G(z)$.

Proposition 1 can be trivially proved on the basis of the intuitive fact that the output response modes of a linear system excited by a step input can be attenuated to an arbitrary extent by filtering through a low-pass filter with sufficiently low bandwidth.
Proposition 1 shows that by properly filtering a square wave input, which is a sequence of steps, we can constrain the input of the backlash nonlinearity to be as close as desired to the output of the filter $F(z)$ scaled by the steady-state gain of the linear system. In turn, this imply that, by properly filtering the square wave input we can constrain the input of the backlash $x_t$ not to overshoot its steady-state value since the step response of the filter $F(z)$ does not overshoot its steady-state value by construction.

Assuming that a rough lower bound $\underline{\omega}_b$ of the system bandwidth is known, a signal $x_t$ without overshoot can be guaranteed by setting $a = \tilde{a}$ such that the bandwidth of the filter is significantly lower than $\underline{\omega}_b$. However, the choice $a = \tilde{a}$ can lead to quite a conservative design of the filter. Since for each different amplitude of the square-wave input one needs to wait until the filter reaches the steady-state operating condition before collecting the output sample, a conservative filter can lead to a time-consuming backlash parameters identification procedure. In order to avoid this problem, the bisection-like search procedure, reported in the Appendix, is proposed to maximize the filter bandwidth. The key step of the procedure is the condition (expressed in line 4.) to test if the designed filter provides a signal $x_t$ without overshoot. As a matter of fact each filter which provides a signal $x_t$ without overshoot leads to the same steady-state value of the backlash output.

*Remark 1* — The proposed search procedure is based on the assumption of using a first order filter. However, the same approach can be used to design a $n$-order filter of the form $F(z) = (1-a)^n/(z-a)^n$.

### *3.2 Steady-state experiment*

In this subsection a few details about the steady-state experiment are presented. For each value of the input square wave amplitude, only one steady-state value of the noisy output is considered on the positive half-wave of the input and one steady-state value of the noisy output on the negative half-wave. Thus, given a set of square wave inputs with $M$ different amplitudes, $2M$ steady-state values of the output are taken into account. Indeed, combining equations (2), (3) and (4) at steady-state, we get the following input-output description involving only the parameters of the backlash:

$$\bar{y}_i = m_r(\bar{u}_i - c_r) + \bar{\eta}_i, \quad \text{for } \bar{u}_i \geq \frac{\bar{x}_{i-1}}{m_r} + c_r \quad i = 1, \ldots, M; \tag{11}$$

$$\bar{y}_j = m_l(\bar{u}_j + c_l) + \bar{\eta}_j, \quad \text{for } \bar{u}_j \leq \frac{\bar{x}_{j-1}}{m_l} - c_l \quad j = 1, \ldots, M; \tag{12}$$

where the triplets $\{\bar{u}_i, \bar{y}_i, \bar{\eta}_i\}$ and $\{\bar{u}_j, \bar{y}_j, \bar{\eta}_j\}$ are collections of steady-state values of the known input signal, output observation and measurement error collected during the positive and the negative square wave respectively. A block diagram description of the steady-state response is depicted in Fig. 3 for equation (11) only; equation (12) leads to a similar block diagram representation. Since the pairs $(m_l, c_l)$ and $(m_r, c_r)$ affect the collected measurements, i.e. equations (11) and (12), separately, we can define the feasible parameter region of the backlash as

$$\mathcal{D}_\gamma = \mathcal{D}_\gamma^r \bigcup \mathcal{D}_\gamma^l \tag{13}$$

where

$$\mathcal{D}_\gamma^r = \{m_r, c_r \in R^+ : \bar{y}_i = m_r(\bar{u}_i - c_r) + \bar{\eta}_i, \\ \bar{\eta}_i \in [-\Delta\bar{\eta}_i - \alpha_i, \Delta\bar{\eta}_i]; \ i = 1, \ldots, M\} \tag{14}$$

$$\mathcal{D}_\gamma^l = \{m_l, c_l \in R^+ : \bar{y}_j = m_l(\bar{u}_j + c_l) + \bar{\eta}_j, \\ \bar{\eta}_j \in [-\Delta\bar{\eta}_j - \alpha_j, \Delta\bar{\eta}_j]; \ j = 1, \ldots, M\} \tag{15}$$

where $\{\Delta\bar{\eta}_i\}$ and $\{\Delta\bar{\eta}_j\}$ are the sequences of bounds on measurements uncertainty, $\alpha_i = 2\Delta\bar{\eta}_i\bar{u}_i/\bar{r}_F$, $\alpha_j = 2\Delta\bar{\eta}_j\bar{u}_j/\bar{r}_F$ and $\bar{r}_F$ is the steady-state value of the step input used in the experiment performed to tune the filter pole $a$. From definition (13) it can be seen that $\mathcal{D}_\gamma$ is exactly described by the following constraints in the parameter space

$$\bar{y}_i - m_r(\bar{u}_i - c_r) \geq -\Delta\bar{\eta}_i - \alpha_i, \quad \bar{y}_i - m_r(\bar{u}_i - c_r) \leq \Delta\bar{\eta}_i, \\ m_r > 0, c_r > 0, \qquad i = 1, \ldots, M \tag{16}$$

$$\bar{y}_j - m_l(\bar{u}_j + c_l) \geq -\Delta\bar{\eta}_j - \alpha_j, \quad \bar{y}_j - m_l(\bar{u}_j + c_l) \leq \Delta\bar{\eta}_j, \\ m_l > 0, c_l > 0, \qquad j = 1, \ldots, M \tag{17}$$

### *3.3 Outer-bounding Orthotope description of $\mathcal{D}_\gamma$*

An exact description of $\mathcal{D}_\gamma$ can be given in terms of edges, each one being described, from a practical point of view, as a subset of an active constraint lying between two vertices. Since $\mathcal{D}_\gamma$ has exactly the same geometrical structure of the feasible parameter set of the backlash nonlinearity considered in Cerone and Regruto [2007], we can exploit the effective procedure proposed in such a paper for deriving active constraints, vertices and edges. Edges provide exact description of $\mathcal{D}_\gamma$ which, on the downside, could be not so easy to handle. A somewhat more practical description, although approximate, can be obtained by the computation of the following orthotope outer-bounding set $\mathcal{B}_\gamma$ tightly containing $\mathcal{D}_\gamma$:

$$\mathcal{B}_\gamma = \{\gamma \in R^4 : \gamma_j = \gamma_j^c + \delta\gamma_j, \mid \delta\gamma_j \mid \leq \Delta\gamma_j, j = 1, \ldots, 4\}, \quad (18)$$

where

$$\gamma_j{}^c = \frac{\gamma_j^{min} + \gamma_j^{max}}{2}, \qquad \Delta\gamma_j = \frac{\mid \gamma_j^{max} - \gamma_j^{min} \mid}{2} \quad (19)$$

$$\gamma_j^{min} = \min_{\gamma \in \mathcal{D}_\gamma} \gamma_j, \quad \gamma_j^{max} = \max_{\gamma \in \mathcal{D}_\gamma} \gamma_j. \quad (20)$$

Since constraints (16) (17) defining $\mathcal{D}_\gamma$ are nonlinear in $\gamma$, at least in principle the solution of the above optimization problems (20) requires the use of nonconvex optimization techniques which, however, do not guarantee the finding of the global optimal solution. Problems (20) can be solved to global optimum thanks to the result reported in Cerone and Regruto [2007].

## 4. INNER SIGNAL BOUNDS EVALUATION

In Section 3 an uncertain description of the backlash is obtained exploiting steady-state data. In order to estimate the parameters of the linear model in the third stage, one should first evaluate the inner signal $x_t \in R$ from the output records $y_t$ of a dynamic experiment. Unfortunately, one must consider the fact that the backlash nonlinear characteristic is in general noninvertible, which means that, given the measured output $y_t$, the signal $x_t$ cannot be evaluated uniquely, giving rise to possible nonconsistent inner signal estimates. Given the estimated uncertain backlash nonlinearity $\mathcal{N}(\gamma)$ and a sequence of measured outputs $\{y_t\}$, obtained exciting the system under consideration with a suitable exciting input sequence $\{u_t\}$, in this section it is shown how upper and lower bounds on the samples of the unmeasurable inner signal $x_t$ can be evaluated.

The solution to the above described problem given in this paper is based on the following ideas: (a) the backlash input cannot be uniquely determined when the backlash output samples exhibit constant values (which means that the input lies somewhere in the deadzone although it is not known precisely where) and (b) given a suitable exciting input sequence $\{u_t\}$, only a set of output sequences without consecutive constant samples are considered for inversion in order to estimates the inner signal $x_t$.

It is pointed out that the dynamic experiment has been carried out using a PRBS input, which, thanks to its nice properties, is successfully employed in linear system identification (Ljung [1999], Söderström and Stoica [1989]). Although PRBS inputs are not suitable for nonlinear system identification in general (Bai [2002], Ninness and Gibson [2002]) since it may not adequately excite the unknown nonlinearity, in Bai [2004] it is shown that such a signal can be effectively used to decouple the linear and nonlinear parts in the identification of Hammerstein model with static nonlinearity. In paper Cerone and Regruto [2007] it is shown that the use of a PRBS sequence is profitable for the identification of linear system with input backlash. The following definitions and results are given.

*Definition 1.* $Y \subset R$, also called the set of *Invertible Output Sequence*, is the set of $y_t \in R$ for which the backlash input $x_t$ can be uniquely determined, i.e.:

$$Y = \{y_t \in R : x_t \leftrightarrow y_t\} \quad (21)$$

*Proposition 2.* An output sample $y_t$ belongs to the set of Invertible Output Sequence $Y$ if and only if $|y_t - y_{t-1}| > 2\Delta\eta$.

*Definition 2.* $X \subset R$, also called the set of *Feasible Inner-signal Sequence*, is the set obtained through the inversion of the set of the Invertible Output Sequence $Y$, i.e.:

$$X = \left\{x_t \in R : x_t = \mathcal{N}^{-1}(y_t, \gamma), y_t \in Y\right\} \quad (22)$$

*Proposition 3.* Given the estimated Backlash nonlinearity $\mathcal{N}(x_t, \gamma)$ with $\gamma \in \mathcal{D}_\gamma$, a PRBS input $\{u_t\}$ and the sequence of output $y_t \in Y$, each sample $x_t$ of the set of the Feasible Inner-signal Sequence $X$ is bounded as follows:

$$x_t^{min} \leq x_t \leq x_t^{max} \quad (23)$$

$$x_t^{max} = \max_{|\eta_t| \leq \Delta\eta, \ \gamma \in \mathcal{D}_\gamma} \frac{y_t - \eta_t}{\gamma_1} + \gamma_2 \quad \text{if } y_t < y_{t-1} - 2\Delta\eta \quad (24)$$

$$x_t^{max} = \max_{|\eta_t| \leq \Delta\eta, \ \gamma \in \mathcal{D}_\gamma} \frac{y_t - \eta_t}{\gamma_3} + \gamma_4 \quad \text{if } y_t > y_{t-1} + 2\Delta\eta \quad (25)$$

$$x_t^{min} = \min_{|\eta_t| \leq \Delta\eta, \ \gamma \in \mathcal{D}_\gamma} \frac{y_t - \eta_t}{\gamma_1} + \gamma_2 \quad \text{if } y_t < y_{t-1} - 2\Delta\eta \quad (26)$$

$$x_t^{min} = \min_{|\eta_t| \leq \Delta\eta, \ \gamma \in \mathcal{D}_\gamma} \frac{y_t - \eta_t}{\gamma_3} + \gamma_4 \quad \text{if } y_t > y_{t-1} + 2\Delta\eta \quad (27)$$

*Proposition 4.* The global optimal solutions of problems (24) – (27) occur on the vertices of $\mathcal{D}_\gamma$ and for $\eta = \pm\Delta\eta$.

## 5. BOUNDING THE PARAMETERS OF THE LINEAR DYNAMIC MODEL

In this section bounds $\theta_j^{max}$ and $\theta_j^{min}$ on each parameter of the linear dynamic block are evaluated. Defining the quantities

$$x_t^c = \frac{x_t^{min} + x_t^{max}}{2}, \qquad \Delta x_t = \frac{x_t^{max} - x_t^{min}}{2} \quad (28)$$

the following relation can be established between the unknown inner signal $x_t$ and the central value $x_t^c$:

$$x_t = x_t^c + \delta x_t \quad (29)$$

$$\mid \delta x_t \mid \leq \Delta x_t. \quad (30)$$

The identification of the linear block can be formulated in the frame of output error models, i.e., in terms of the known input sequence $\{u_t\}$ and the uncertain inner sequence $\{x_t\}$ as shown in Figure 4. This stage of the procedure is quite standard and it will not be discussed in the paper. The interested readers can find the details in the previous works by the authors Cerone and Regruto [2003], Cerone and Regruto [2007] and Cerone [1993].

Applying results from Cerone [1993] to the considered

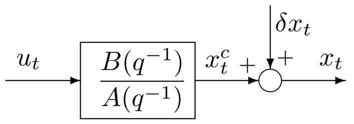


Fig. 4. Output error set-up for bounding the parameters of the linear system.

problem uncertainty bounds $\Delta\theta_j = \frac{|\theta_j^{max} - \theta_j^{min}|}{2}$ and central estimates $\theta_j^c = \frac{\theta_j^{min} + \theta_j^{max}}{2}$ are computed for each parameter $\theta_j$.

## 6. A SIMULATED EXAMPLE

In this section we illustrate the proposed parameter bounding procedure through a numerical example. The system considered here is characterized by a linear block with $A(q^{-1}) = (1 + 0.5q^{-1} + 0.7q^{-2})$ and $B(q^{-1}) = (1.4q^{-1} + 0.8q^{-2})$ and a nonsymmetric backlash with $m_l = 0.248$, $m_r = 0.252$, $c_l = 0.0698$, $c_r = 0.0349$. Thus, the true parameter vectors are $\gamma = [m_l \; c_l \; m_r \; c_r]^{\mathrm{T}} = [0.248 \;\; 0.0698 \;\; 0.252 \;\; 0.0349]^{\mathrm{T}}$ and $\theta = [a_1 \;\; a_2 \;\; b_1 \;\; b_2]^{\mathrm{T}} = [0.5 \; 0.7 \;\; 1.4 \;\; 0.8]^{\mathrm{T}}$. We emphasize that the backlash parameters have been realistically chosen: as a matter of fact we considered the parameters of a real world precision gearbox which features a gear ratio equal to 0.25 and a deadzone as low as 0.0524 rad ($\approx 3^o$) and simulated a possible fictitious nonsymmetric backlash with gear ratio $m_l = 0.248$, $m_r = 0.252$ and deadzone $c_l = 0.0698$ ($\approx 4^o$), $c_r = 0.0349$ ($\approx 2^o$). Bounded absolute output errors have been considered when simulating the collection of both steady state data, $\{\bar{u}_s, \bar{y}_s\}$, and transient sequence $\{u_t, y_t\}$. Here we assumed $|\eta_t| \leq \Delta\eta_t$ and $|\bar{\eta}_s| \leq \Delta\bar{\eta}_s$ where $\eta_t$ and $\bar{\eta}_s$, are random sequences belonging to the uniform distributions $U[-\Delta\eta_t, +\Delta\eta_t]$ and $U[-\Delta\bar{\eta}_s, +\Delta\bar{\eta}_s]$ respectively. Bounds on steady-state and transient output measurement errors were supposed to have the same value, i.e., $\Delta\eta_t = \Delta\bar{\eta}_s \doteq \Delta\eta$. Different values of $\Delta\eta$ were chosen in such a way as to simulate the measurement errors of commercial absolute binary encoder with a number of bits $n_{bit}$ varying from 8 to 15. For a given $\Delta\eta$, the length of steady-state and the transient data are $M = 50$ and $N = [100]$ respectively. The value $a = 0.8125$ of the filter pole has been set using the procedure described in the Appendix. The steady-state input samples $\bar{u}_s$ are equally spaced values from 0.6 and 3, while the transient input sequence $\{u_t\}$ is a PRBS which takes the values $\pm 1$. Results about the nonlinear and the linear block are reported in Table I and Tables II respectively. For low noise level ($n_{bit} > 10$ bits) and for all $N$, the central estimates of both the nonlinear static block and the linear model are consistent with the true parameters. For higher noise levels ($n_{bit} \leq 10$ bits), both $\gamma^c$ and $\theta^c$ give satisfactory estimates of the true parameters.

## 7. CONCLUSION

A three-stage parameter bounding procedure for linear systems with output backlash in presence of bounded output errors has been outlined. The proposed approach is based on suitable strategies for the decoupling of the linear and nonlinear blocks. More specifically, in the first stage the backlash parameters are estimated exciting the system with a properly filtered square wave, while in the second stage inner-signal bounds are computed through a suitable inversion of the backlash nonlinearity. Once such inner-signal bounds have been computed, linear system parameters bounds are evaluated by means of standard results.

## APPENDIX

**Search procedure** (Design of filter pole $a$)

1. Set $a_U = \tilde{a}$, $a_L = 0$, $a^* = a_L$.
2. Set *tol* and $\varepsilon$.
3. Compute $w_s^{\tilde{a}} =$ steady-state value of the output with filter pole $\tilde{a}$ when the input $r_t$ of the filter is a step signal of amplitude $\bar{r}_F$.
4. Compute $w_s^{a^*} =$ steady-state output of the system with filter pole $a^*$ when the input $r_t$ of the filter is a step signal of amplitude $\bar{r}_F$.
5. If $|w_s^{\tilde{a}} - w_s^{a^*}| \leq \varepsilon$ then
6. $\quad a_U = a^*$,
7. else
8. $\quad a_L = a^*$
9. end if
10. If $|a_U - a_L| \geq$ tol then
11. $\quad a^* = \frac{a_U + a_L}{2}$
12. $\quad$ goto 3.
13. end if
14. $a = a^*$
15. end.

*Remark* — In presence of output measurement noise with bounds $\pm\Delta\eta$, step 4 of the proposed search procedure becomes $|y_s^{\tilde{a}} - y_s^{a^*}| \leq 2\Delta\eta + \varepsilon$.

## REFERENCES


E.W. Bai. An optimal two-stage identification algorithm for Hammerstein-Wiener nonlinear systems. *Automatica*, 34(3):333–338, 1998.

E.W. Bai. Identification of linear systems with hard input nonlinearities of known structure. *Automatica*, 38:853–860, 2002.

E.W. Bai. Decoupling the linear and nonlinear parts in Hammerstein model identification. *Automatica*, 40(4): 671–676, 2004.

C.N. Bapat, N. Popplewell, and K. McLachlan. Stable periodic motions of an impact-pair. *Journal of Sound and Vibration*, 87:19–40, 1983.

S.A. Billings. Identification of nonlinear systems — a survey. *IEE Proc. Part D*, 127(6):272–285, 1980.

V. Cerone. Feasible parameter set for linear models with bounded errors in all variable. *Automatica*, 29(6):1551–1555, 1993.

V. Cerone and D. Regruto. Parameter bounds for discrete time Hammerstein models with bounded output errors. *IEEE Trans. Automatic Control*, 48(10):1855–1860, 2003.

V. Cerone and D. Regruto. Bounding the parameters of linear systems with input backlash. *IEEE Trans. Automatic Control*, 52(3):531–536, 2007.

V. Cerone, D. Piga, and D. Regruto. Parameter bounds evaluation for linear system with output backlash. *DAUIN Technical Report, DAI09CPR01*, 2009.

M.L. Corradini, G. Orlando, and G. Parlangeli. A VSC approach for the robust stabilization of nonlinear plants with uncertain nonsmooth actuator nonlinearities — A unified framework. *IEEE Trans. Automatic Control*, 49 (5):807–813, 2004.

A. Krzyżak. Identification of nonlinear block-oriented systems by the recursive kernel estimate. *Int. J. Franklin Inst.*, 330(3):605–627, 1993.

Z.Q. Lang. Controller design oriented model identification method for Hammerstein system. *Automatica*, 29(3): 767–771, 1993.

Z.Q. Lang. A nonparametric polynomial identification algorithm for the Hammerstein system. *IEEE Trans. Automatic Control*, 42(10):1435–1441, 1997.

L. Ljung. *System Identification, Theory for the User.* Prentince Hall, Upper Saddle River, 1999.

M. Milanese and A. Vicino. Optimal estimation theory for dynamic sistems with set membership uncertainty: an overview. *Automatica*, 27(6):997–1009, 1991.

K.S. Narenda and P.G. Gallman. An iterative method for the identification of nonlinear systems using a Hammerstein model. *IEEE Trans. Automatic Control*, AC-11: 546–550, 1966.

B. Ninness and S. Gibson. Quantifying the accuracy of Hammerstein model estimation. *Automatica*, 38:2037–2051, 2002.

M. Nordin and P.Õ. Gutman. Controlling mechanical systems with backlash — a survey. *Automatica*, 38: 1633–1649, 2002.

T. Söderström and P. Stoica. *System Identification.* Prentice Hall, Upper Saddle River, 1989.

P. Stoica and T. Söderström. Instrumental-variable methods for identification of Hammerstein systems. *Int. J. Control*, 35(3):459–476, 1982.

L. Sun, W. Liu, and A. Sano. Identification of a dynamical system with input nonlinearity. *IEE Proc. Part D*, 146 (1):41–51, 1999.

G. Tao and C.A. Canudas de Wit (Eds.). Special issue on adaptive systems with non-smooth nonlinearities. *Int. J. of Adapt. Control & Sign. Proces.*, 11(1), 1997.

G. Tao and P.V. Kokotovic. *Adaptive control of systems with actuator and sensor nonlinearities.* Wiley, New York, NY, 1996.

E. Walter and H. Piet-Lahanier. Estimation of parameter bounds from bounded-error data: a survey. *Mathematics and Computers in simulation*, 32:449–468, 1990.

T. Wigren. Output error convergence of adaptive filters with compensation for output nonlinearities. *IEEE Trans. Automatic. Control*, 43(7):975–978, 1998.


Table I: Nonlinear block parameter central estimates $\gamma^c$ and parameter uncertainty bounds $\Delta\gamma$ against varying number of bits ($n_{bit}$).

| $n_{bit}$ | $\gamma_j$ | True Value | $\gamma_j^c$ | $\Delta\gamma_j$ |
|---|---|---|---|---|
| 15 | $m_l$ | 0.2480 | 0.2481 | 1.1e-4 |
| | $c_l$ | 0.0698 | 0.0702 | 4.8e-4 |
| | $m_r$ | 0.2520 | 0.2520 | 2.1e-5 |
| | $c_r$ | 0.0349 | 0.0349 | 1.0e-4 |
| 12 | $m_l$ | 0.2480 | 0.2474 | 6.6e-4 |
| | $c_l$ | 0.0698 | 0.0678 | 2.4e-3 |
| | $m_r$ | 0.2520 | 0.2521 | 1.3e-4 |
| | $c_r$ | 0.0349 | 0.0354 | 7.0e-4 |
| 10 | $m_l$ | 0.2480 | 0.2503 | 2.8e-3 |
| | $c_l$ | 0.0698 | 0.0825 | 1.6e-2 |
| | $m_r$ | 0.2520 | 0.2513 | 8.0e-4 |
| | $c_r$ | 0.0349 | 0.0306 | 5.2e-3 |
| 8 | $m_l$ | 0.2480 | 0.2475 | 2.9e-3 |
| | $c_l$ | 0.0698 | 0.0655 | 1.8e-2 |
| | $m_r$ | 0.2520 | 0.2615 | 1.1e-2 |
| | $c_r$ | 0.0349 | 0.0625 | 3.7e-2 |

Table II: Linear system parameter central estimates ($\theta_j^c$) and parameter uncertainty bounds ($\Delta\theta_j$) against varying number of bits ($n_{bit}$) and signal to noise ratio (SNR) when $N = 100$.

| $n_{bit}$ | SNR (dB) | $\theta_j$ | True Value | $\theta_j^c$ | $\Delta\theta_j$ |
|---|---|---|---|---|---|
| 15 | 73.2 | $\theta_1$ | 0.500 | 0.500 | 4.3e-4 |
| | | $\theta_2$ | 0.700 | 0.700 | 2.5e-4 |
| | | $\theta_3$ | 1.400 | 1.400 | 4.3e-4 |
| | | $\theta_4$ | 0.800 | 0.800 | 7.2e-4 |
| 12 | 56.2 | $\theta_1$ | 0.500 | 0.500 | 4.8e-3 |
| | | $\theta_2$ | 0.700 | 0.700 | 2.8e-3 |
| | | $\theta_3$ | 1.400 | 1.400 | 8.7e-3 |
| | | $\theta_4$ | 0.800 | 0.800 | 1.0e-2 |
| 10 | 43.0 | $\theta_1$ | 0.500 | 0.503 | 1.8e-2 |
| | | $\theta_2$ | 0.700 | 0.698 | 1.9e-2 |
| | | $\theta_3$ | 1.400 | 1.400 | 4.2e-2 |
| | | $\theta_4$ | 0.800 | 0.806 | 4.6e-2 |
| 8 | 30.9 | $\theta_1$ | 0.500 | 0.510 | 6.9e-2 |
| | | $\theta_2$ | 0.700 | 0.706 | 5.3e-2 |
| | | $\theta_3$ | 1.400 | 1.410 | 1.1e-1 |
| | | $\theta_4$ | 0.800 | 0.805 | 1.2e-1 |