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LPV identification of the glucose-insulin dynamics in Type I diabetes

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Abstract: In this paper we address the problem of identifying a linear parameter varying (LPV) model of the glucose-insulin dynamics in Type I diabetic patients. First, the identification problem is formulated in the framework of bounded-error identification, then an algorithm for parameter bounds computation, based on semidefinite programming, is presented. The effectiveness of the proposed approach is tested in simulation by means of the widely adopted nonlinear Sorensen patient model.

Keywords: Bounded-error identification, LPV modeling, Type I diabetes.

1. INTRODUCTION

Patients affected by Type I diabetes mellitus suffer from a metabolic disorder characterized by pancreas inability to produce a sufficient amount of insulin, a chronic condition which leads to incorrect regulation of blood glucose concentration. As a consequence, food intake results in a blood glucose level significantly higher than the upper limit of the so called *normoglycemic range* ($60 - 120$ mg/dL), a dangerous medical condition. According to the 2011 national statistic report of the USA Department of health [USA National Diabetes Information Clearinghouse] (2011), diabetes is the leading cause of kidney failure, nontraumatic lower-limb amputations and blindness among adults, a major cause of heart disease and stroke, and the seventh leading cause of death in the United States.

Since traditional medical treatment of Type I diabetes, based on multiple daily subcutaneous insulin injections, has proved to be inadequate in many cases, significant research efforts have been devoted in the last three decades towards the realization of an artificial pancreas, i.e. a feedback control system for automatic real-time regulation of the blood glucose concentration. The key ingredient of this system, which makes use of small size implantable glucose sensors and programmable insulin pumps to physically close the loop, is the control algorithm aimed at performing real-time computation of the insulin delivering strategy. A remarkable number of approaches for blood glucose control have appeared in the literature in the last decade including classical PID control (Steil et al. (2004)), optimal control (Fisher and Teo (1989)), sliding mode control (Abu-Rmileh et al. (2010)), robust H_∞ control (Parker et al. (2000); Ruiz-Velázquez et al. (2004)) and model predictive control (Lee and Bequette (2009); Dua et al. (2006); Magni et al. (2009)) just to cite a few. An extensive critical review of available blood glucose control algorithms can be found in the paper (Bequette (2005)).

Although most of the compartmental/physiologically-inspired models proposed in the literature to describe the glucose-insulin dynamics in Type I diabetic patients are nonlinear (see, e.g., Sorensen (1985); Dalla Man et al. (2007)), most of the available control techniques mentioned above are based on approximated linear models obtained by linearization. In fact, although the models proposed in (Sorensen (1985); Dalla Man et al. (2007)) are proven to be useful for accurate simulation and detailed analysis of the glucose-insulin dynamics in Type I diabetic patients, their complexity prevent, in practice, their use for nonlinear controller design. Furthermore, the large number of physiological parameters involved in these patient mathematical descriptions, makes it difficult to efficiently take into account the interpatient variability in the controller design, and tuning of the model to a specific patient seems not be an easy task.

A possible way to deal with the problem of balancing the trade-off between model complexity and model accuracy, is to resort to the linear parameter varying (LPV) modeling paradigm which is now considered one of the most effective tool to derive mathematical description of nonlinear/time-varying phenomena (see, e.g., the book by Tóth (2010)). Since LPV models are linear systems whose parameters depend on time-varying exogenous variables, whose real-time measurements are assumed to be available, they can be considered the bridge between the simple linear-time-invariant (LTI) models and the general nonlinear ones. Based on this fact, a number of contributions has appeared in the last two decades where results of optimal and robust LTI control theory are extended to the LPV framework (see, e.g., Packard (1994); Becker and Packard (1994); Apkarian and Gahinet (1995); Wu et al. (1996); Leith and Leithead (2000); Shamma (1991); Scherer (2001)). In particular, an interesting design technique has been recently proposed in Gilbert et al. (2010) that allows the designer to arbitrary select the LPV controller order and test if there exist a controller of

such an order which solve the problem, overcoming one of the major limitation of the previously proposed algorithms.

In this paper we address the problem of identifying an LPV model of the glucose-insulin dynamics in the input-output form suitable for the application of the controller design technique proposed in Gilbert et al. (2010). The proposed LPV identification technique, which is based on the set-membership identification theory, provides a tool to systematically derive a mathematical model tailored to the specific patient, overcoming the important problem of interpatient variability. To the authors' best knowledge, this is the first attempt to apply LPV identification technique to the problem of Type I diabetic patient modeling. The paper is organized as follows. Set-membership LPV identification theory is briefly reviewed in Section 2. In Section 3, a black-box LPV model in the input-output form suitable for the LPV controller design proposed in Gilbert et al. (2010) is derived from a set of data generated by simulating the Sorensen nonlinear patient model (Sorensen (1985)). Concluding remarks end the paper.

2. SET-MEMBERSHIP LPV IDENTIFICATION

Consider the following multi-input single-output (MISO) discrete-time LPV model described in the I/O form:

$$\mathcal{A}(q^{-1},\lambda(t))w(t) = \mathcal{B}(q^{-1},\lambda(t))u(t), \tag{1a}$$
$$\mathcal{A}(q^{-1},\lambda(t))\eta(t) = \mathcal{D}(q^{-1},\lambda(t))e(t), \tag{1b}$$
$$y(t) = w(t) + \eta(t), \tag{1c}$$
$$z(t) = \lambda(t) + \varepsilon(t), \tag{1d}$$

where q^{-1} is the backward time-shift operator, *i.e.*, $q^{-1}w(t) = w(t-1)$, $u(t) = [u^{(1)}(t),\ u^{(2)}(t),\ \ldots,\ u^{(n_u)}(t)] : \mathbb{Z} \to \mathbb{R}^{n_u}$ is the vector of input signals, $w(t) : \mathbb{Z} \to \mathbb{R}$ is the noise-free output signal, $y(t) : \mathbb{Z} \to \mathbb{R}$ is the measured output signal, $e(t) : \mathbb{Z} \to \mathbb{R}$ is a bounded noise, $\eta(t) : \mathbb{Z} \to \mathbb{R}$ is the effect of the noise $e(t)$ on the measured output signal, $\lambda(t) : \mathbb{Z} \to \mathbb{R}$ is the scheduling variable which, according to the LPV modeling and control literature (see, *e.g.*, Rugh and Shamma (2000)) is assumed to be measurable, $z(t) : \mathbb{Z} \to \mathbb{R}$ is the measured scheduling signal, while $\varepsilon(t) : \mathbb{Z} \to \mathbb{R}$ is a bounded noise corrupting the scheduling signal measurements. The model structure (1a-d) is a quite general one. In fact the ARX model structure is obtained by setting $\mathcal{D} = 1$, the output-error (OE) is given by the choice $\mathcal{D} = \mathcal{A}$, while the case $\mathcal{A} \neq \mathcal{B} \neq \mathcal{D}$ leads to the ARMAX structure, see Tóth (2010) for details.

In the rest of the paper we will use the shorthand notation π_t to denote the generic signal $\pi(t)$. Note that, since the case of multiple input is considered, $\mathcal{B}(\cdot)$ is a vector of functions, i.e. $\mathcal{B}(\cdot) = \left[\mathcal{B}^{(1)}(\cdot),\ldots,\mathcal{B}^{(n_u)}(\cdot)\right]$. In this work we assume that $\mathcal{A}(\cdot)$, each element of vector $\mathcal{B}(\cdot)$, and $\mathcal{D}(\cdot)$ are polynomials in the backward shift operator q^{-1} which depend nonlinearly on the scheduling variables according to the following equations

$$\mathcal{A}(q^{-1},\lambda_t) = 1 + a_1(\lambda_t)q^{-1} + \ldots + a_{n_\mathrm{a}}(\lambda_t)q^{-n_\mathrm{a}}, \tag{2a}$$
$$\mathcal{B}^{(k)}(q^{-1},\lambda_t) = b_1^{(k)}(\lambda_t)q^{-1} + \ldots + b_{n_\mathrm{b}}^{(k)}(\lambda_t)q^{-n_\mathrm{b}}, \tag{2b}$$
$$k = 1,\ldots,n_u,$$
$$\mathcal{D}(q^{-1},\lambda_t) = d_0(\lambda_t) + d_1(\lambda_t)q^{-1} + \ldots + d_{n_\mathrm{d}}(\lambda_t)q^{-n_\mathrm{d}}, \tag{2c}$$

where $n_\mathrm{a}, n_\mathrm{b}, n_\mathrm{d} \geq 0$ and the coefficients a_i, $b_j^{(k)}$ and d_l are assumed to be static functions of λ_t, parameterized in the form

$$a_i(\lambda_t) = \sum_{s=0}^{n_{\phi,i}} a_{i,s}\phi_{i,s}(\lambda_t), \quad b_j^{(k)}(\lambda_t) = \sum_{s=0}^{n_{\psi,j}} b_{j,s}^{(k)}\psi_{j,s}(\lambda_t), \tag{3a}$$

$$d_l(\lambda_t) = \sum_{s=0}^{n_{\sigma,l}} d_{l,s}\sigma_{l,s}(\lambda_t) \tag{3b}$$

where $\phi_{i,s}(\cdot)$, $\psi_{j,s}(\cdot)$ and $\sigma_{l,s}(\cdot)$ are a-priori chosen nonlinear functions belonging to the canonical polynomial basis in the parameters λ_t $(1,\ \lambda_t,\ \lambda_t^2,\ \lambda_t^3,\ldots)$.

According to the *set-membership framework*, e_t and ε_t are only assumed to range within given bounds Δe and $\Delta\varepsilon$ respectively, i.e.

$$|e_t| \leq \Delta e, \tag{4}$$
$$|\varepsilon_t| \leq \Delta\varepsilon. \tag{5}$$

The unknown parameter vector $\theta \in \mathbb{R}^{n_\theta}$ to be estimated is defined as

$$\theta = \big[a_{1,0}\ \ldots\ a_{1,n_{\phi,1}}\ \ldots\ a_{n_\mathrm{a},1}\ \ldots\ a_{n_\mathrm{a},n_{\phi,n_\mathrm{a}}}\ b_{1,0}^{(1)}\ \ldots\ b_{n_\mathrm{b},n_{\psi,n_\mathrm{b}}}^{(1)}\ \ldots\ b_{1,0}^{(n_u)}\ \ldots\ b_{n_\mathrm{b},n_{\psi,n_\mathrm{b}}}^{(n_u)}\ \ldots\ d_{n_\mathrm{d},n_{\sigma,n_\mathrm{d}}}\big]. \tag{6}$$

with n_θ denoting the number of parameters to be estimated. In the context of set-membership identification, all parameter values consistent with collected measurements, a-priori information on system and error bounds are feasible solutions of the identification problem. This set, $\mathcal{P}_\theta \subset \mathbb{R}^{n_\theta}$, commonly called feasible parameter set, is the projection on the parameter space of the set $\mathcal{P}$ given by (1a-d), giving

$$\mathcal{P} = \Big\{(\theta,\eta,\varepsilon) \in \mathbb{R}^{n_\theta}\times\mathbb{R}^N\times\mathbb{R}^N : \mathcal{A}(q^{-1},z_t-\varepsilon_t)y_t = \mathcal{B}(q^{-1},z_t-\varepsilon_t)u_t + \mathcal{D}(q^{-1},z_t-\varepsilon_t)e_t,\ |e_t| \leq \Delta e,\ |\varepsilon_t| \leq \Delta\varepsilon,\ t = 1,\ldots,N\Big\}, \tag{7}$$

where N is the length of the data sequence. Tight bounds on each component θ_j of the parameter vector can be computed, on the basis of the set $\mathcal{P}$, by solving the following optimization problems

$$\underline{\theta}_j = \min_{(\theta,\eta,\varepsilon)\in\mathcal{P}} \theta_j, \quad \overline{\theta}_j = \max_{(\theta,\eta,\varepsilon)\in\mathcal{P}} \theta_j. \tag{8}$$

The computed bounds, which implicitly define the parameter uncertainty interval $PUI_{\theta_j} = [\underline{\theta}_j\ \overline{\theta}_j]$, can be used to compute the so-called central estimate θ^c of the parameter vector θ, defined as:

$$\theta^c = [\theta_1^c \ldots \theta_{n_\theta}^c]^\mathrm{T} \tag{9}$$

where

$$\theta_j^c = \frac{\underline{\theta}_j + \overline{\theta}_j}{2},\ j = 1,\ldots,n_\theta. \tag{10}$$

The parameter vector θ^c is the Chebyshev center in the ℓ_∞ norm of $\mathcal{P}_\theta$ and enjoys peculiar optimality properties (see Kacewicz et al. (1986) for details).

It is worth noting that $\mathcal{P}$ is a nonconvex set since its definition involves constraints $\mathcal{A}(q^{-1},z_t-\varepsilon_t)[y_t-\eta_t] = \mathcal{B}(q^{-1},z_t-\varepsilon_t)u_t$ that are polynomial functions in variables θ, η and ε. As a consequence, problems (8) are nonconvex and, therefore, standard nonlinear optimization tools (gradient method, Newton method, etc.) can not be used because they can trap in local minima/maxima. As a consequence, the PUI_j obtained using these tools is not guaranteed to contain the true unknown parameter θ_j, which is a key requirement of any set-membership identification method. One possible solution to overcome such

a problem is to relax identification problems (8) to convex optimization problems in order to numerically compute lower bounds of $\underline{\theta}_j$ as well as upper bounds of $\overline{\theta}_j$. Effective techniques have recently been proposed in (Cerone and Regruto (2008); Cerone et al. (2011)) to compute relaxed bound on θ_j by means of suitable algorithms requiring the solution to a set of convex semidefinite optimization problems.

3. LPV MODELING OF GLUCOSE-INSULIN DYNAMICS

The identification scheme described in the previous section is used to identify an LPV model of glucose-insulin dynamics from a set a data generated by simulating the patient behavior through the nonlinear Sorensen model. The input signals $u_t^{(1)}$ and $u_t^{(2)}$ of the Sorensen model are, respectively, the glucose absorption from the meal intake and the insulin infusion rate, while the output w_t is the blood glucose concentration that is assumed to be collected by a realistic sensor for continuous glucose monitoring, which is reasonably expected to acquire the value of the blood glucose concentration with a sampling time of 5 minutes. The output data w_t are then corrupted by a random additive noise η_t, i.e. $y_t = w_t + \eta_t$, uniformly distributed between $[-\Delta\eta; \Delta\eta] = [-4; 4]$ mg/dL. The chosen value of the error bound $\Delta\eta$ corresponds to the current resolution of sensors for continuous glucose monitoring (see, e.g., Caduff et al. (2003)). The model to be identified is an OE LPV described by eqs. (1a)-(1d) with $\mathcal{A}(q^{-1}) = \mathcal{D}(q^{-1})$, input signals $u_t^{(1)}$ and $u_t^{(2)}$ and noise-corrupted output $y_t = w_t + \eta_t$. The glucose concentration w_t is also used as scheduling parameter λ_t, i.e. $\lambda_t = w_t$ or equivalently $z_t = y_t$. Roughly speaking, such an LPV model can be seen as a infinite collection of linear systems, each of them associated with a different value of the glucose concentration. It is worth remarking that, since λ_t is equal to the output signal w_t, also the scheduling parameter is affected by noise. Therefore, an identification approach which is able to handle also the error on the scheduling parameter should be preferred. To the best of our knowledge, the only method available in literature for LPV identification with error on the scheduling parameters is the one described in the previous section, which relies on the results presented in Cerone and Regruto (2008); Cerone et al. (2011) .

3.1 Data description

The data are collected by simulating the patient behavior for two consecutive days. Each day, the patient is supposed to eat three meals throughout the day (e.g. breakfast, lunch and dinner), six hours apart. The amount of carbohydrate consumed in each meal is reported in Table 1. The meal model proposed in Lehmann and Deutsch (1992) is used to simulate the glucose absorption rate, whose time-domain evolution during the two considered days is plotted in Fig. 1. As far as the insulin infusion rate $u_t^{(2)}$ is concerned, initially $u_t^{(2)}$ is a white random process uniformly distributed between the basal value 22 mU/min, corresponding to the euglycemic equilibrium of the Sorensen model, and 70 mU/min, while $u_t^{(2)}$ is set to the basal value after that the effect of the dinner vanishes and the patient is supposed to sleep (see Fig. 2). The evolution of the blood glucose concentration w_t obtained by simulating the Sorensen model is depicted in Fig. 3. It is worth remarking that a random profile of the insulin infusion rate might not be the optimal signal to be applied in practice to human patients. Nevertheless, the injected insulin levels guarantee that Hypoglycemia (blood glucose concentration below 60 mg/dL) never occurs in the patient. Design of a *human friendly* identification experiment is currently under investigation, while results in this direction are reported in Lee and Bequette (2009) for the identification of LTI patient model.

Table 1. Amount of carbohydrate consumed in the meals.

	Breakfast	Lunch	Dinner
Day 1	50 g	100 g	50 g
Day 2	65 g	90 g	35 g

The data collected in the first day are used to identify the LPV model of the patient, whose performance is tested on a validation set composed of the data collected during the second day. In order to evaluate the matching between the noise-corrupted measured glucose concentration y_t and the estimated one $\hat{y}_t$, we consider the root mean square errors (RMSE) in the identification data set (RMSE_{id}) and in the validation data set (RMSE_{val}). The generic RMSE is defined as

$$RMSE = \sqrt{\sum_{t=1}^{N} \frac{(y_t - \hat{y}_t)^2}{N}}, \tag{11}$$

where N denotes the length of the data set.

3.2 Selection of the LPV model structure and obtained results

Here, the problem of selecting the structure of polynomials $\mathcal{A}(\cdot)$ and $\mathcal{B}(\cdot)$ given in eqs. (1a)-(1b) arises. Indeed, as the degree n_a and n_b of polynomials $\mathcal{A}(\cdot)$ and $\mathcal{B}(\cdot)$ increases, the degrees of freedom of the LPV model increases too, providing a better matching between real data and estimated data in the identification set. However, a model with a large number of degrees of freedom could overfit the data in the identification set, leading to possibly low accuracy of the identified model when tested on the validation data set. Besides, as n_a and n_b increases, the complexity of the identified model increases too. The same considerations hold as far as the choice of the degree of polynomials $a_i(\lambda)$ and $b_j^{(k)}(\lambda)$ in (3a) is concerned. On these basis, we have selected the structure of $\mathcal{A}(\cdot)$ and $\mathcal{B}(\cdot)$ by progressively increasing the values of n_a and n_b and the degree of polynomials $a_i(\lambda)$ and $b_j^k(\lambda)$ until the identified model provides a satisfactory accuracy level on the validation set. The LPV model patient has been identified for ten different structures, whose features are summarized in Table 2.

Table 2. Structures of functions $\mathcal{A}(\cdot)$ and $\mathcal{B}(\cdot)$.

Structure	n_a	n_b	degree of $a_i(\lambda)$	degree of $b_j^{(k)}(\lambda)$
S1	1	1	1	1
S2	2	1	1	1
S3	2	2	1	1
S4	3	2	1	1
S5	3	3	1	1
S6	1	1	2	2
S7	2	1	2	2
S8	2	2	2	2
S9	3	2	2	2
S10	3	3	2	2

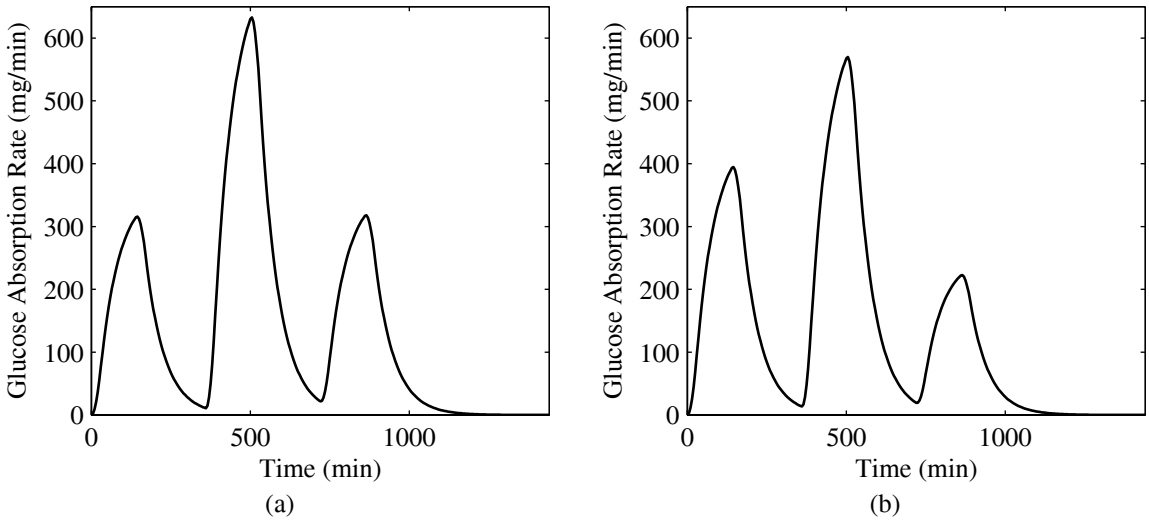

Fig. 1. Glucose absorption rate during the first day (a) and during the second day (b).

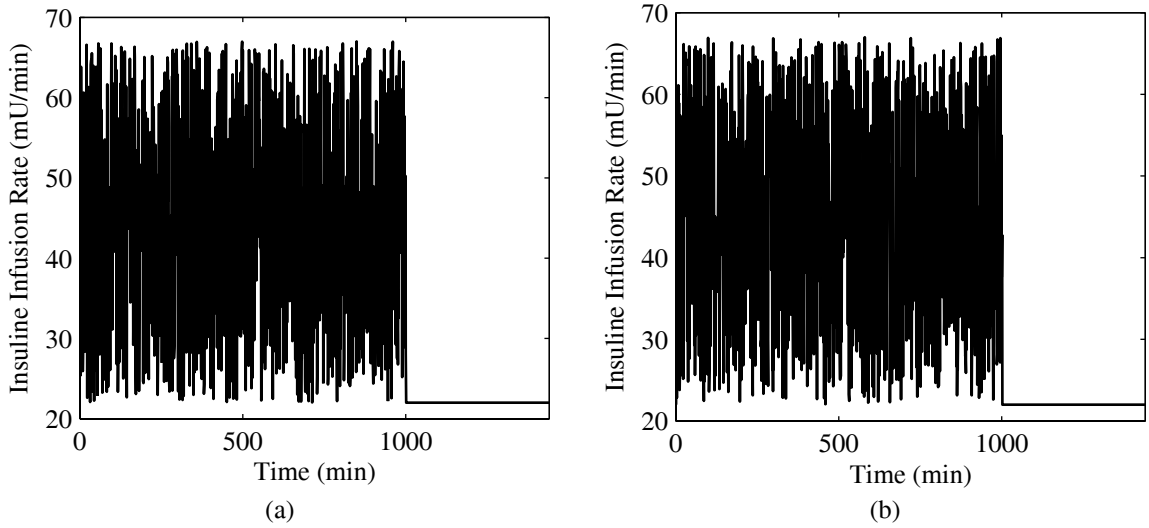

Fig. 2. Insulin rate input during the first day (a) and during the second day (b).

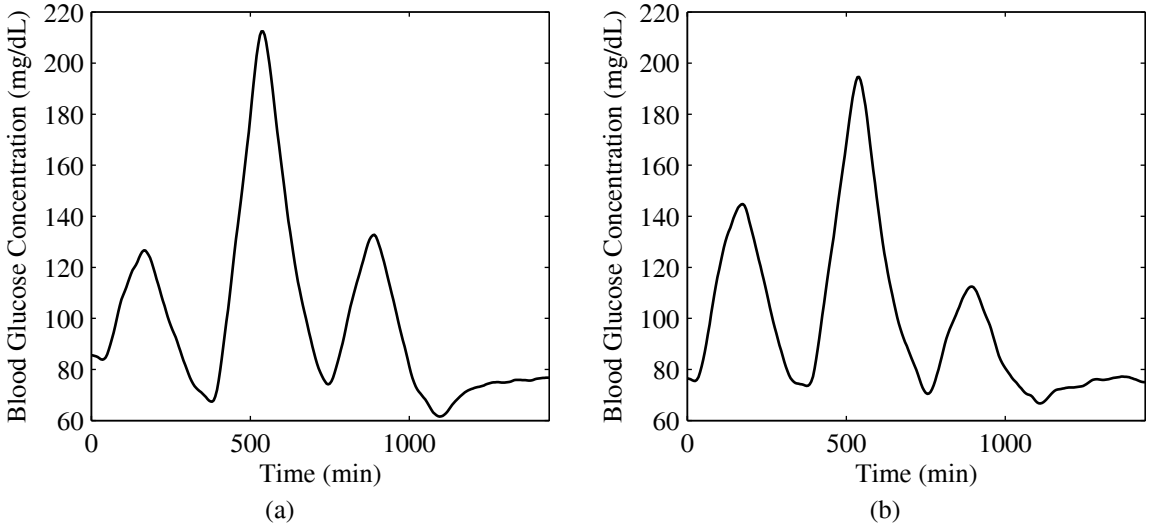

Fig. 3. Blood glucose concentration in the patient during the first day (a) and during the second day (b).

Table 3. Number of estimated parameters (n_θ) and root mean square error RMSE_{val} obtained for structures S1-S10 of functions $\mathcal{A}(\cdot)$ and $\mathcal{B}(\cdot)$.

Structure	RMSE_{val} (mg/dL)	n_θ
S1	11.32	6
S2	8.19	8
S3	5.79	12
S4	5.74	14
S5	5.62	18
S6	10.15	9
S7	7.02	12
S8	5.65	18
S9	5.62	21
S10	5.46	27

On the basis of the collected data, the feasible parameter set $\mathcal{P}_\theta$ is sought. Among all possible parameters belonging to $\mathcal{P}_\theta$, the Chebyshev center defined in (10) and computed through the method proposed in Cerone and Regruto (2008), is chosen as output of the identification algorithm. Table 3 reports the number of estimated parameters of the identified LPV patient model for the structures S1-S10 described in Table 2, together with the root mean square errors RMSE_{val} in the validation set. By analyzing results in Table 3, we have chosen the structure S3 to describe the glucose-insulin dynamics of the patient. Such an LPV structure provides a good tradeoff between accuracy and complexity of the model. Therefore, the mapping between the output w_t and input signals $u_t^{(1)}$ and $u_t^{(2)}$ is given by

$$\mathcal{A}(q^{-1},\lambda_t)w_t = \mathcal{B}^{(1)}(q^{-1},\lambda_t)u_t^{(1)} + \mathcal{B}^{(2)}(q^{-1},\lambda_t)u_t^{(2)}, \quad (12)$$

with

$$\mathcal{A}(q^{-1},\lambda_t) = 1 + (a_{1,0} + a_{1,1}\lambda_t)\,q^{-1} + (a_{2,0} + a_{2,1}\lambda_t)\,q^{-2}, \quad (13)$$

$$\mathcal{B}^{(1)}(q^{-1},\lambda_t) = \left(b_{1,0}^{(1)} + b_{1,1}^{(1)}\lambda_t\right)q^{-1} + \left(b_{2,0}^{(1)} + b_{2,1}^{(1)}\lambda_t\right)q^{-2}, \quad (14)$$

$$\mathcal{B}^{(2)}(q^{-1},\lambda_t) = \left(b_{1,0}^{(2)} + b_{1,1}^{(2)}\lambda_t\right)q^{-1} + \left(b_{2,0}^{(2)} + b_{2,1}^{(2)}\lambda_t\right)q^{-2}. \quad (15)$$

The computed values of the identified LPV parameters are $[a_{1,0},\ a_{1,1},\ a_{2,0},\ a_{2,1}] = [-0.3975,\ 0.0009,\ -0.1439,\ -0.0020]$, $[b_{1,0}^{(1)},\ b_{1,1}^{(1)},\ b_{2,0}^{(1)},\ b_{2,1}^{(1)}] = [-0.1680,\ 0.0013,\ 0.1833,\ -0.0012]$ and $[b_{1,0}^{(2)},\ b_{1,1}^{(2)},\ b_{2,0}^{(2)},\ b_{2,1}^{(2)}] = [1.1584,\ -0.0068,\ -0.0941,\ 0.0081]$. It is worth remarking that the computed values of parameters $a_{1,1}, a_{2,1}, b_{1,1}^{(1)}, b_{2,1}^{(1)}, b_{1,1}^{(2)}, b_{2,1}^{(2)}$ multiplying the scheduling variable λ_t are small with respect to the values of $a_{1,0}, b_{1,0}^{(1)}, b_{2,0}^{(1)}, b_{1,0}^{(2)}, b_{2,0}^{(2)}$. Therefore, one should think that the effect of parameters $a_{1,1}, a_{2,1}, b_{1,1}^{(1)}, b_{2,1}^{(1)}, b_{1,1}^{(2)}, b_{2,1}^{(2)}$ is negligible and so they can be set to zero in order to work with an LTI system instead of an LPV model. However, that is not correct. In fact, since the scheduling parameter λ_t takes values from 60 mg/dL up to 220 mg/dL, the product between $a_{1,1}, a_{2,1}, b_{1,1}^{(1)}, b_{2,1}^{(1)}, b_{1,1}^{(2)}, b_{2,1}^{(2)}$ and λ_t is comparable with the values of parameters $a_{1,0}, b_{1,0}^{(1)}, b_{2,0}^{(1)}, b_{1,0}^{(2)}, b_{2,0}^{(2)}$.

The comparison between the noise-free blood glucose concentration w_t obtained by simulating the patient behavior through the Sorensen model and the glucose concentration simulated by the identified LPV model is reported in Fig. 4, which shows a good matching between the outputs of the two systems.

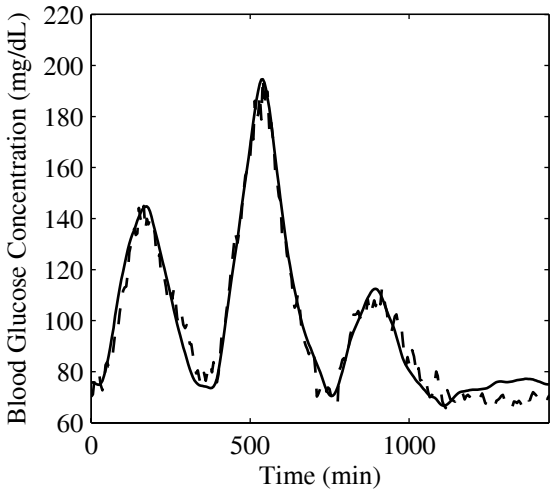

Fig. 4. Comparison between blood glucose concentration obtained by simulating the patient behavior through the Sorensen model (solid line) and estimated output of the identified LPV patient model (dashed line).

4. CONCLUSIONS

Identification of a linear-parameter varying (LPV) model describing glucose-insulin dynamics in a patient affected by Type I diabetes is addressed in the paper. A set-membership approach to the identification of LPV systems, when both the output and the scheduling parameter measurements are affected by bounded noise is exploited to identify an LPV patient model from a set of data obtained by simulating the widely adopted nonlinear Sorensen model. The proposed approach is computationally tractable since it requires the solution to simple linear programming problems. The identified model provides a satisfactory description of glucose-insulin dynamics in a form suitable for the design of an LPV controller for automatic regulation of the blood glucose concentration, which is the subject of ongoing research.

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