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Exploiting the Shapley value in the estimation of the position of a point of interest for a group of individuals

Antonio Camurri[a], Floriane Dardard[a,b], Simone Ghisio[a], Donald Glowinski[a], Giorgio Gnecco[a,c], Marcello Sanguineti[a,*]

[a] *DIBRIS - University of Genoa, Via Opera Pia 13, 16145 Genova, Italy*
[b] *Département d'Informatique École Normale Supérieure - 45 Rue Ulm, Paris, France*
[c] *IMT - Institute for Advanced Studies - Piazza S. Ponziano 6 - 55100 Lucca, Italy*

Abstract

Concepts and tools from cooperative game theory are exploited to quantify the role played by each member of a team in estimating the position of an observed point of interest. The measure of importance known as "Shapley value" is used to this end. From the theoretical point view, we propose a specific form of the characteristic function for the class of cooperative games under investigation. In the numerical analysis, different configurations of a group of individuals are considered: all individuals looking at a mobile point of interest, one of them replaced with an artificially-generated one who looks exactly toward the point of interest, and directions of the heads replaced with randomly-generated directions. The corresponding experimental outcomes are compared.

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Keywords: group behavior, cooperative games, Shapley value, point of interest

1. Introduction

The role played by the member of a team in reaching a common goal can be investigated exploiting tools and methods from cooperative game theory (see, e.g., [1]). Starting from the work of Penrose in 1946, devoted to majority voting, several "measures of importance", known as *power indices* were proposed and investigated to model different contexts; see, e.g., [2, 3, 4, 5].

In this paper, we focus on the power index called *Shapley value* [3]. We investigate the possibility of exploiting it to quantify the role played by each member of a team in estimating the position of a point of interest observed by the members themselves. In the experiments, conducted at Casa Paganini - InfoMus Research Centre in Genova, Italy (`www.infomus.org/index_eng.php`), first the behaviors of the participants have been measured by means of the Qualisys Motion Capture (MoCap) system, then a first elaboration of the data has been made by the Qualisys Track Manager software, and further elaborations have been performed by using the EyesWeb XMI (eXtended Multimodal Interaction) software platform (`www.eyesweb.org`) and MATLAB 7.7. We focus on a particular component of the recordings, namely, the time-series data of the positions of the heads of the team members. We measure the position and direction of the head of each team member by means of three markers: two placed above the eyes and one on the back of the head. The position of a point of interest is recorded, too. The members of the team are individuals who have been asked to look at a marker situated at a mobile point. In the experimental analysis, we

*Corresponding author
Email address: `antonio.camurri@unige.it`, `floriane.dardard@ens.fr`, `ghisio@infomus.org`, `donald.glowinski@unige.it`, `giorgio.gnecco@imtlucca.it`, `giorgio.gnecco@unige.it`, `marcello.sanguineti@unige.it` (Marcello Sanguineti)

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compare the results obtained under different configurations: 1) all the members have been instructed to look at a mobile point of interest; 2) one of them is replaced with an artificially-generated one, who looks exactly toward the point of interest; 3) the directions of the members' heads are replaced with randomly-generated directions.

We consider for the team behavioral features related to the displacement of members' heads with respect to a specific *point of interest (PoI)*, to which the team members have been instructed to focus their attention in the first experiment. In order to estimate the position of such a point, we use the directions of the heads instead of the eye-gaze, which, in principle, are better suited. A reason for such a choice is the fact that eye-tracking equipment is still nowadays intrusive and costly. Moreover, previous studies have shown that head direction and eye-gaze are often correlated and have been already exploited to estimate the position of a common point of interest of a group of people, e.g., in [6, 7, 8, 9].

The focus of the paper consists in defining a suitable measure - based on cooperative game theory - of the individual contribution of each team member in determining an estimate of the position of the point of interest. In the first step of our analysis such an estimate is obtained - without resorting to cooperative game theory - by using head positions and directions. In the second step, cooperative game theory - more specifically, the Shapley value and its normalization - is exploited to evaluate the individual contributions to the position estimate. To this end, for the class of cooperative games under investigation, we propose a specific form of the characteristic function of the game, which figures in the definition of the Shapley value and its normalization. The proposed approach for the choice of the characteristic function is then validated by numerical tests.

Preliminary results based on the use of methods from Game Theory and, more generally, Operations Research, to study expressivity of non-verbal social signals in small groups of users were presented at the conference [10], where a team whose members are players in an orchestra was considered as a test-bed.

The remaining of the paper is organized as follows. In Section 2, behavioral features of the team are described. Section 3 is devoted to illustrate the approach based exploiting the Shapley value to estimate the contribution of each team member in the determination of the position of the point of interest. Section 4 presents the results obtained in three different experimental setups. Section 5 is a conclusive discussion, with pointers to possible directions of research.

2. Description of the implemented behavioral features

In this section, we detail the features implemented to characterize group behavior. The behaviors of the participants to the experiment was captured by means of the Qualisys Motion Capture (MoCap) system (Qualisys, Gothenburg, Sweden). Seven cameras were used. Various reflective markers were placed on the body of each person. More specifically, in the present study we used the three markers placed on their heads. Two markers were placed on the forehead and one on the back of the inion. The data were recorded with the Qualisys Track Manager software. A real-time application based on the EyesWeb XMI (eXtended Multimodal Interaction) software platform (`www.eyesweb.org`) was developed to synchronize the 7-cameras Qualisys MoCap data together with video recordings. Further elaborations of the data were performed by using MATLAB 7.7.

The team members are numbered from 1 to N. For each recording, the frames are identified by the index $k = 1, \ldots, N_{\text{frames}}$.

2.1. Convergence of the head directions toward the point of interest

We denote by $\mathbf{p}_{PoI}^{(k)}$ the position vector of the point of interest. The following procedure was followed, for each frame k of each recording.

1) For each team member i ($i = 1, \ldots, N$), we compute the current position vector $\mathbf{p}_i^{(k)}$ in the horizontal[1] plane of the member's head center of mass as the mean of the position vectors describing the three markers located on the head (assuming the same mass for each marker). Then define the current direction $\mathbf{d}_i^{(k)}$ in the horizontal plane of the head of the member i as the unit vector connecting the center of mass of his/her head

[1] We consider only the horizontal components of the head movement, so all vectors are 2-dimensional. The reason is that the vertical displacements of the two markers placed above the eyes of each member may be misleading in estimating the three-dimensional head direction, since the markers are not placed on the eyes but above them.

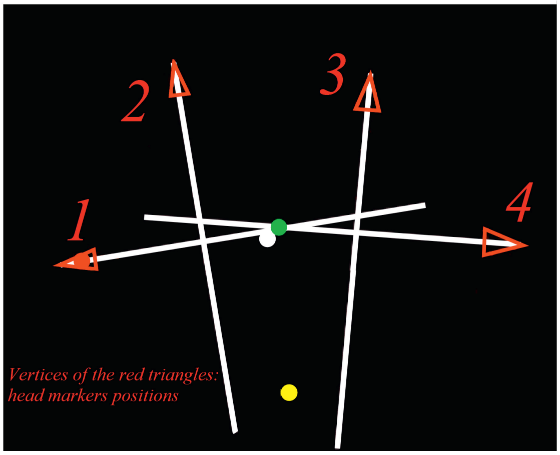

Fig. 1. Snapshot of the head markers positions of the team members for $N = 4$. The white half-lines refer to the head directions and the green dot corresponds to the position of the *point of convergence of the team* (*PoCT*) (i.e., the point to which all the head directions converge), which is also the *point of convergence* PoC_S associated with the subset (*subteam*) $S = \{1, 2, 3, 4\}$ of team members (numbered from left to right). The yellow dot represents the position of the point of convergence PoC_S associated with $S = \{2, 3, 4\}$. The position of the *point of interest PoI* of the team is illustrated by the white dot. Finally, the red point represents the position of the center of mass of the head of the first member of the team.

to the point located in the middle of the line between the two markers above his eyes (see Figure 1).

2) For each member i ($i = 1, \ldots, N$), we consider the half-line $\mathbf{HL}_i^{(k)}$ starting from the point $\mathbf{p}_i^{(k)}$ and with direction $\mathbf{d}_i^{(k)}$, i.e., the set of all the points with position vectors

$$\boxed{\mathbf{p}_i^{(k)} + t\mathbf{d}_i^{(k)}\,,} \tag{1}$$

where $t \geq 0$ is any nonnegative real number.

3) For each pair (i, j) of members ($i, j = 1, \ldots, N, i < j$), we compute the position $\mathbf{p}_{i,j}^{(k)}$ of the intersection between the two half-lines $\mathbf{HL}_i^{(k)}$ and $\mathbf{HL}_j^{(k)}$. As $\mathbf{p}_i^{(k)} \neq \mathbf{p}_j^{(k)}$, such an intersection exists if and only if the following condition is met:

– the algebraic linear system (in the real unknowns u and v)

$$\boxed{\mathbf{p}_i^{(k)} + u\mathbf{d}_i^{(k)} = \mathbf{p}_j^{(k)} + v\mathbf{d}_j^{(k)}} \tag{2}$$

has a unique solution (this happens if and only if $\mathbf{d}_i^{(k)}$ is not parallel to $\mathbf{d}_j^{(k)}$) and the obtained values of u and v are nonnegative.

When the condition above holds, the position vector $\mathbf{p}_{i,j}^{(k)}$ is equivalently defined as

$$\boxed{\mathbf{p}_{i,j}^{(k)} = \mathbf{p}_i^{(k)} + u\mathbf{d}_i^{(k)}\,,} \tag{3}$$

or

$$\boxed{\mathbf{p}_{i,j}^{(k)} = \mathbf{p}_j^{(k)} + v\mathbf{d}_j^{(k)}\,.} \tag{4}$$

The procedure is repeated $\frac{N(N-1)}{2}$ times, determining - for the frames for which they exist - the $\frac{N(N-1)}{2}$ position vectors $\mathbf{p}_{i,j}^{(k)}$ of the $\frac{N(N-1)}{2}$ pairwise intersections[2].

[2]E.g., for $N = 4$, one obtains the six position vectors $\mathbf{p}_{1,2}^{(k)}, \mathbf{p}_{1,3}^{(k)}, \mathbf{p}_{1,4}^{(k)}, \mathbf{p}_{2,3}^{(k)}, \mathbf{p}_{2,4}^{(k)}, \mathbf{p}_{3,4}^{(k)}$.

4) For each subset (*subteam*) $S \subseteq \{1, \ldots, N\}$ of team members, we denote by $I_S^{(k)}$ the subset of the pairs (i, j), with $i, j \in S, i < j$, for which the pairwise intersections above exist at frame k, and by $|I_S^{(k)}|$ its cardinality. If $I_S^{(k)}$ is nonempty, then the position vector at frame k of the *point of convergence* (PoC_S) *associated with the subteam* S - the point where the directions of all subteam members' head directions belonging to S converge (see Figure 1) - is defined as the position of the center of mass of the intersection points belonging to $I_S^{(k)}$ (assuming the same mass for each intersection point):

$$\boxed{\mathbf{p}_{PoC_S}^{(k)} = \frac{\sum_{(i,j)\in I_S^{(k)}} \mathbf{p}_{i,j}^{(k)}}{|I_S^{(k)}|}\,.} \tag{5}$$

If $I_S^{(k)}$ is empty, then the point of convergence is not defined at frame k. When $S = \{1, \ldots, N\}$, we call the associated point *point of convergence of the team*, and denote it by $PoCT$.

5) For each subset $S \subseteq \{1, \ldots, N\}$, we evaluate the Euclidean distance $\|\mathbf{p}_{PoC_S}^{(k)} - \mathbf{p}_{PoI}^{(k)}\|$ between the point of convergence associated with S and the point of interest, when the former point exists.

One can interpret the position of PoC_S as an estimate of the position of the point of interest of the team, obtained from the behavior of the team members belonging to S. Note that, for $|S| > 1$, when the PoC_S and all the points of convergence $PoC_{S\setminus\{i\}}$ ($i \in S$) exist at frame k, one has the recursive relation[3]

$$\boxed{\mathbf{p}_{PoC_S}^{(k)} = \frac{\sum_{i\in S} \mathbf{p}_{PoC_{S\setminus\{i\}}}^{(k)}}{|S|}\,,} \tag{6}$$

which means that in such a case PoC_S is the center of mass of the system of points of convergence $PoC_{S\setminus\{i\}}$, where $i \in S$ (again, assuming the same mass for each such point).

2.2. *Degrees of uncertainty of the points of convergence*

Another interesting feature is the *degree of uncertainty* $DoU_{PoC_S}^{(k)}$, at the generic frame k, of the point of convergence PoC_S associated with the subteam $S \subseteq \{1, \ldots, N\}$, defined as

$$\boxed{DoU_{PoC_S}^{(k)} := \begin{cases} 0, & \text{if } |I_S^{(k)}| = 0, 1\,, \\ \sqrt{\frac{1}{|I_S^{(k)}|} \sum_{(i,j)\in I_S^{(k)}} \|\mathbf{p}_{i,j}^{(k)} - \mathbf{p}_{PoC_S}^{(k)}\|^2}, & \text{if } |I_S^{(k)}| > 1\,. \end{cases}} \tag{7}$$

For $|I_S^{(k)}| = 0, 1$, the degree of uncertainty of the point of convergence associated with the subteam S is equal to 0: indeed, the point of convergence is not defined for $|I_S^{(k)}| = 0$, and for $|I_S^{(k)}| = 1$ it coincides with the unique intersection point defining it. Instead, when $|I_S^{(k)}| > 1$, the degree of uncertainty is the root-mean-squared distance of such point of convergence from the pairwise intersection points from which it is defined. Loosely speaking, a small value of the degree of uncertainty means that the location of each pairwise intersection in $I_S^{(k)}$ is a good estimate of the position of the point of convergence associated with the subteam S (which, in turn, is an estimate of the position of the point of interest, obtained from the subteam S). Instead, a large value means that one cannot conclude that all the heads of the subteam members are really directed toward such point. In other words, the position of the point of convergence PoC_S for the subteam S is expected to be a more reliable estimate of the location of the point of interest PoI of the team when the value of its degree of uncertainty DoU_{PoC_S} is smaller.

The point of convergence associated with each subset S and its degree of uncertainty have the following geometrical and physical interpretations. First, being defined as the center of mass of suitable pairwise intersections between the head directions, the point of convergence for S lies on the convex hull of such points. In particular, the smaller the perimeter and the area of such convex hull, the smaller the value of DoU_{PoC_S}. The quantity

$$\boxed{\sum_{(i,j)\in I_S^{(k)}} \|\mathbf{p}_{i,j}^{(k)} - \mathbf{p}_{PoC_S}^{(k)}\|^2} \tag{8}$$

[3] Slightly different relations are obtained when some of the points of convergence $PoC_{S\setminus\{i\}}$ ($i \in S$) do not exist at frame k.

in (7) can be interpreted as the moment of inertia of the system of such pairwise intersection points with respect to a vertical axis passing through the PoC_S at frame k, assuming the same unit mass for each pairwise intersection point. The quantity

$$\boxed{\frac{1}{|I_S^{(k)}|}\sum_{(i,j)\in I_S^{(k)}} \|\mathbf{p}_{i,j}^{(k)} - \mathbf{p}_{PoC_S}^{(k)}\|^2} \tag{9}$$

can be interpreted as the normalization of such a moment with respect to the total mass $|I_S^{(k)}|$ of the system. So, a small value of the degree of uncertainty is associated with a small value of the normalized moment of inertia and, vice-versa, a large value of the degree of uncertainty has to do with a large value of the normalized moment of inertia.

2.3. Mean angles between the head directions and the half-lines joining each head to the point of interest

Let us denote by $\theta_i^{(k)}$ the angle at frame k between the head direction of the member i and the half-line connecting his/her head to the point of interest. By definition, such an angle lies between 0 and π rad. An interesting feature (exploited in the following, together with other features, to evaluate the utility of each subteam S) is the mean $\theta_S^{(k)}$ of $\theta_i^{(k)}$ with respect to the members of each subteam S, which we call *mean angle of semi-aperture* associated with the subteam S:

$$\boxed{\theta_S^{(k)} = \begin{cases} 0, & \text{for } |S| = 0\,, \\ \frac{\sum_{i\in S}\theta_i^{(k)}}{|S|}, & \text{for } |S| \geq 1\,. \end{cases}} \tag{10}$$

This angle lies between 0 and π rad, too, and is equal to 0 in the particular case in which the heads of all members of the subteam S are directed exactly toward the point of interest. So, the feature $\theta_S^{(k)}$ quantifies on average how much the head of each member of S is actually directed toward the point of interest in frame k.

3. Shapley values of the team members

We are interested in estimating the contribution of each team member in the determination of the position of the point of interest. We propose to achieve this objective through an application of cooperative game theory [11]. More precisely, we shall compute the *Shapley value* of each member of the team, assuming that the objective of the team consists in estimating - through the determination of the point of convergence of the team, $PoCT$ - the position of the point of interest itself.

Recall that in a cooperative game[4], the Shapley value provides a measure of the "importance" of each player. It is defined as the average marginal contribution of that player to the utility of every possible subteam. In our experiments, let each subteam $S \subseteq \{1,\ldots,N\}$ with cardinality $|S|$ be associated with a *utility* $v^{(k)}(S)$ at frame k (using the terminology of cooperative game theory, the function $v^{(k)}(\cdot)$ is also called *characteristic function*). Then the Shapley value of $i = 1,\ldots,N$ at frame k is defined as [11]

$$\boxed{Sh_i^{(k)} := \frac{\sum_{S\subseteq\{1,\ldots,N\}}\left(\left(v^{(k)}(S) - v^{(k)}(S\setminus\{i\})\right)\frac{(|S|-1)!(N-|S|)!}{N!}\right)}{2^N-1}\,.} \tag{11}$$

In order to apply the Shapley value to the context under examination, we have to assign "reasonable" values $v^{(k)}(S)$ to each subteam $S \subseteq \{1,\ldots,N\}$. In particular, for each frame k of each recording, we propose to define $v^{(k)}(S)$ as follows:

$$\boxed{v^{(k)}(S) = \begin{cases} 0, & \text{for } |S| = 0, 1, \text{ or when } PoC_S \text{ is not defined at frame } k\,, \\ \frac{1}{\varepsilon + \|\mathbf{p}_{PoC_S}^{(k)} - \mathbf{p}_{PoI}^{(k)}\| + DoU_{PoC_S}^{(k)} + K\cdot\theta_S^{(k)}}, & \text{for } |S| > 1 \text{ and } PoC_S \text{ is defined at frame } k\,, \end{cases}} \tag{12}$$

[4]More precisely in a so-called *transferable utility (TU)* cooperative game [11].

where $\varepsilon > 0$ is a suitably small constant, introduced to avoid too large values of $v^{(k)}(S)$, and $K > 0$ is a suitably large constant, which quantifies the trade-off between the terms

$$\|\mathbf{p}^{(k)}_{PoC_S} - \mathbf{p}^{(k)}_{PoI}\| + DoU^{(k)}_{PoC_S}$$

and

$$K \cdot \theta^{(k)}_S .$$

For $|S| = 0, 1$ we let $v^{(k)}(S) = 0$, as an empty set of team members does not provide any estimate of the point of interest and the direction of the head of a single team member alone is not enough to obtain a valid estimate. Otherwise, the utility is equal to the inverse of the sum of ε, the distance $\|\mathbf{p}^{(k)}_{PoC_S} - \mathbf{p}^{(k)}_{PoI}\|$ between the point of convergence associated with S and the point of interest, the degree of uncertainty related to that point of convergence, and the mean angle of semi-aperture for the subteam S, multiplied by K.

One motivation for the introduction of the term $K \cdot \theta^{(k)}_S$ inside the utility of the subteam S is to distinguish between two different situations corresponding to different positions of the heads of the members of the subteam S but for which the pairwise intersections of the directions of such heads are exactly the same (so the term $\|\mathbf{p}^{(k)}_{PoC_S} - \mathbf{p}^{(k)}_{PoI}\| + DoU^{(k)}_{PoC_S}$ is the same in both situations, and it is not able to discriminate between them). Such two different situations may be artificially obtained, e.g., by rigid translations - of sufficiently small sizes - of the head of one member along the head direction. A second motivation for the introduction of the term $K \cdot \theta^{(k)}_S$ is that it penalizes subteams in which one of the angles $\theta^{(k)}_i$ (for some $i \in S$) is so large that there may not exist pairwise intersection points with indexes in $I^{(k)}_S$, in which one of the half-lines is determined by the direction of the head of member i. So, in this case a desired negative contribution of the member i to the utility of the subteam S would not be included in the term $\|\mathbf{p}^{(k)}_{PoC_S} - \mathbf{p}^{(k)}_{PoI}\| + DoU^{(k)}_{PoC_S}$ but it would be included in the term $K \cdot \theta^{(k)}_S$.

It is worth remarking that, in general, the cooperative game with the utility (12) is not superadditive [11], in the following sense: it is not true that, for any pair of subteams S_1, S_2 such that $S_1 \cap S_2 = \emptyset$, one has necessary $v^{(k)}(S_1) + v^{(k)}(S_2) \geq v^{(k)}(S_1 \cup S_2)$. This implies that, for the cooperative game defined by the utility (12), the Shapley value of a team member may be negative.

One can prove (see, e.g., [11]) that

$$\boxed{\sum_{i=1}^{N} Sh^{(k)}_i = v^{(k)}(\{1, \ldots, N\})} \tag{13}$$

and by (12) one has $v^{(k)}(\{1, \ldots, N\}) \geq 0$. So, another interesting quantity is the *normalized Shapley value*

$$\boxed{\texttt{normalized}\ Sh^{(k)}_i = \begin{cases} \frac{Sh^{(k)}_i}{v^{(k)}(\{1,\ldots,N\})}, & \text{if } v^{(k)}(\{1, \ldots, N\}) > 0\,, \\ \frac{1}{N}, & \text{if } v^{(k)}(\{1, \ldots, N\}) = 0\,, \end{cases}} \tag{14}$$

which provides an estimate of the relative importance of each team member in the determination of the utility of the team (notwithstanding the value of the utility of the team itself). Note that, by definition, the sum of the normalized Shapley values with respect to the team members is always equal to 1, although the normalized Shapley values of some team members may be negative when the game is not superadditive.

4. Experimental results

We have implemented the following experimental setups.

- *Condition* 1. A team made up of $N = 4$ members, who have been instructed to look at a mobile point of interest.
- *Condition* 2. One of the 4 members - in the specific case, the 1st one - has been replaced with an artificially-generated one, whose head position is the same of the replaced member but whose head is directed exactly toward the point of interest.
- *Condition* 3. The directions of the heads of all the team members have been replaced with artificial randomly-generated directions, according to uniform distributions for their angles (whereas the positions of the heads have not been changed, compared with Condition 1).

Condition	Median of the utility of the team
1	0.0024
2	0.0033
3	0.0002

Table 1. Median of the utility $v^{(k)}(\{1,\dots,N\})$ of the team $\{1,\dots,N\}$ for $N = 4$, under conditions 1, 2, and 3.

	Median of the normalized Shapley value			
Condition	Team member 1	Team member 2	Team member 3	Team member 4
1	0.0980	0.3026	0.3349	0.3124
2	0.3384	0.2351	0.2074	0.2977
3	0.25	0.25	0.25	0.25

Table 2. Medians of the normalized Shapley values under conditions 1, 2, and 3.

In the data analysis, the medians of suitable features have been considered instead of their averages because they are less sensitive to outliers. In particular, we have estimated:

1. the capability of the team to estimate the location of the point of interest. In the paper, this is evaluated in terms of the median - with respect to all the frames - of the utility of the team $v^{(k)}(\{1,\dots,N\})$;
2. the capability of the normalized Shapley value to quantify the importance of each member in determining the quality of the above-mentioned estimate. The median - with respect to all the frames - of the normalized Shapley value has been evaluated to this aim.

Under condition 2 one expects, with respect to condition 1:

- an increase of the median - with respect to all the frames - of the utility of the team $v^{(k)}(\{1,\dots,N\})$;
- an increase of the median - with respect to all the frames - of the normalized Shapley value of the member who has been replaced (equal to the index of the artificially-generated member).

Moreover, under Condition 3 one expects, with respect to Conditions 1 and 2,

- a reduction of the median - with respect to all the frames - of the utility of the team $v^{(k)}(\{1,\dots,N\})$;
- almost equal values of the medians - with respect to all the frames - of the normalized Shapley values of the members.

The number of frames of the available recording is $N_{\text{frames}} = 4238$. The expectations above have been confirmed by the results, obtained in MATLAB 7.7 and reported in Tables 1 and 2. They show, respectively, the median of the utility of the team and the medians of the normalized Shapley values in the three conditions. Similarly, Figures 2 and 3 show, respectively, the utility of the team for each frame and the normalized Shapley value for a subset of the frames. In Figure 3, for graphical reasons only a subset of the frames has been considered, with the aim of having a sufficiently small variability of the normalized Shapley values on such a subset. By the way, the frames considered in Figure 3 correspond to a situation in which the point of interest was nearly fixed. Figure 2 and Tables 1 and 2, instead, take into account all the frames of the recording. Finally, for the constants inside formula (12) we made the choices $\varepsilon = 10^{-6}$ and $K = 500$. As to the specific recording, the results were only marginally influenced by the choice of ε, whose value was much smaller than the typical values assumed over the frames by the other terms in the second formula (12). Instead, the value of K was chosen sufficiently large in order to have a significant dependence of the second formula (12) on the angular term.

One can observe, e.g., that the improvements above of Condition 1 with respect to Condition 2 refer to median values and do not hold frame-by-frame (although there are improvements over almost all frames). Moreover, note that the obtained curves are not smooth. This may be mainly due to the fact that the cardinalities $|I_S^{(k)}|$ (used in the definitions of the points of convergence and of their degrees of uncertainty) depend on the frame index k.

5. Discussion

The proposed approach, based on the use of the point of convergence of a team as an estimate of the location of a specific point of interest, can be applied to teams of musicians, e.g., the string quartet context and the orchestra context, presented, respectively in [12] and [13]. In the first case, the point of interest may be represented by the location of the conductor or by the position of a particular musician (e.g., the concertmaster). In the second case, the role of point of interest may be played by the so-called *ear* of the quartet (as defined in [14]) or by the location of a particular musician (e.g., the first violinist). These two settings differ from the one considered in Section 4, as they refer to applications in which team members were not instructed beforehand to follow a specific point of common interest (each musician also directs naturally his attention to his music stand). However, as an consequence of synchronization, a point of interest often emerges during particular moments of the performance. In order to apply the tools exploited in this paper to the string quartet and the orchestra contexts, such a point needs to be detected in advance. There are other contexts in which tools of Operations Research have been applied to music; see [15] for a recent survey of such applications.

In this work we have exploited the Shapley value of each member of the team, assuming that the objective of the team consists in estimating - through the determination of the point of convergence of the team PoCT - the position of the point of interest itself. However, we are aware that such an approach has some drawbacks, as it may be inadequate to catch the power of each member in interaction with the others. Such drawbacks may be overcome by combining game theory and graph theory - more specifically, by using graph theory in the design of the characteristic function. This can be done by taking into account application-dependent relationships among the team members, where the relationships and the team members are modeled, respectively, as edges and nodes in a graph. In the present context, the geometric configuration of the team may be modeled by weighted edges. A possible direction of research that combines game theory and graph theory consists in exploiting power indices for *communication structures* [16]. For instance, possible relations among the members can be modeled by means of the communication structure first proposed by Myerson [17]; then, the Shapley value of a suitable restricted game can be evaluated. Using the concepts of *interaction network* and *gene-k-gene situation*, originally developed for biological networks [18], the role of the various members can be estimated with respect to their capability to act as “bridges” between pairs of other members. Finally, when the interactions among members have a probabilistic nature, the theory developed for values of *games with probabilistic graphs* [19], which extends to a stochastic interaction context the original theory of Myerson [17], may represent a useful approach.

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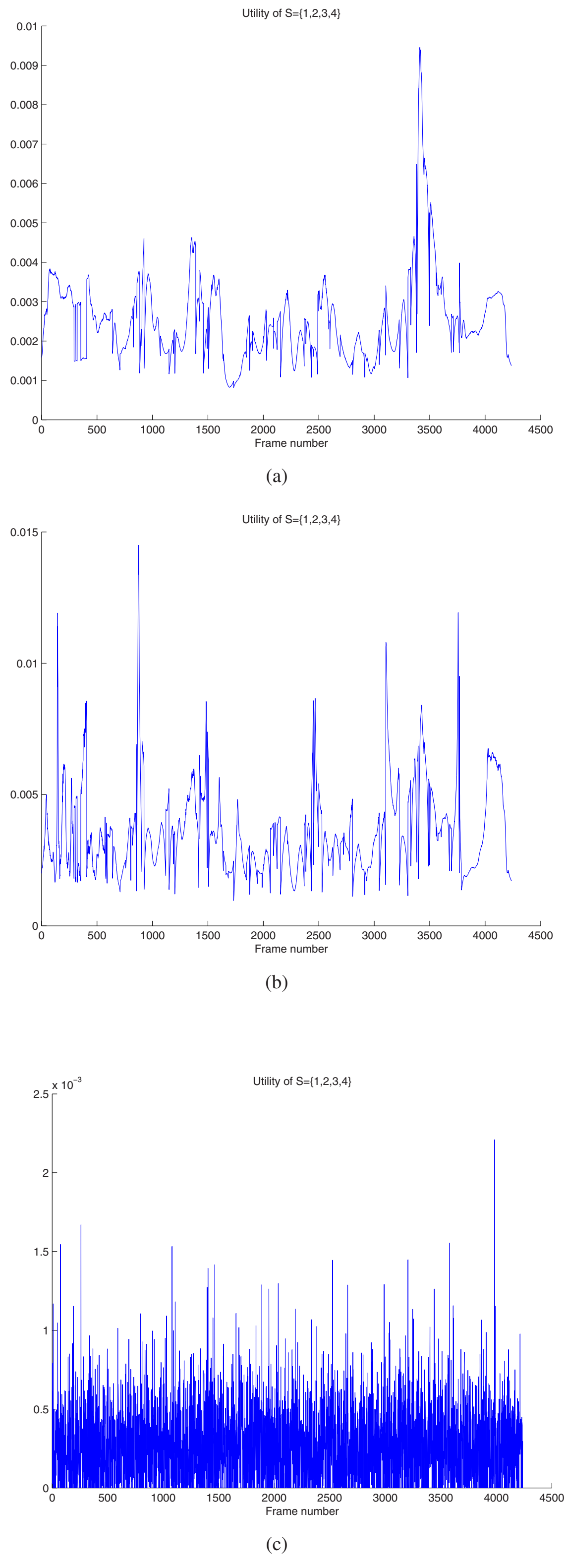
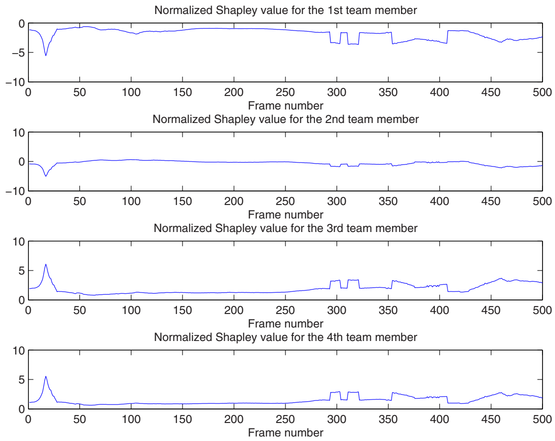


(a)

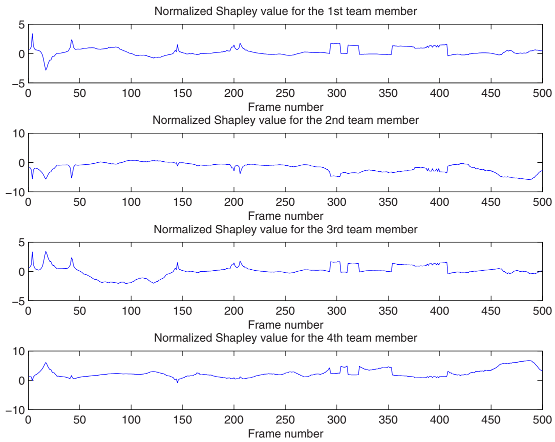
(b)

(c)

Fig. 2. Utility $v^{(k)}(\{1,\ldots,N\})$ of the team $\{1,\ldots,N\}$ for $N=4$, under conditions 1 (a), 2 (b), and 3 (c).



(a)



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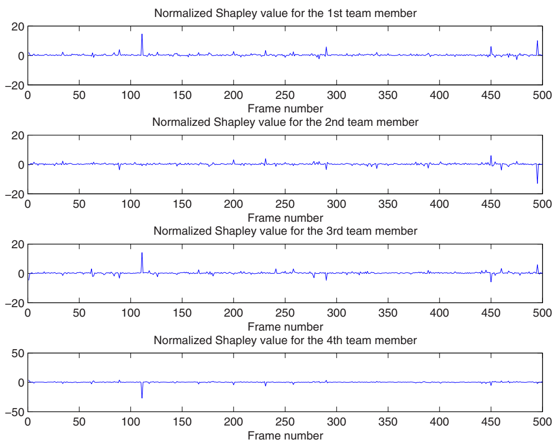

(c)

Fig. 3. Normalized Shapley values under conditions 1 (a), 2 (b) and 3 (c), for a subset of the frames of the available recording.