

# Distress propagation in complex networks: the case of non-linear DebtRank

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We consider a dynamical model of distress propagation on complex networks, which we apply to the study of financial contagion in networks of banks connected to each other by direct exposures. The model that we consider is an extension of the DebtRank algorithm, recently introduced in the literature. The mechanics of distress propagation is very simple: When a bank suffers a loss, distress propagates to its creditors, who in turn suffer losses, and so on. The original DebtRank assumes that losses are propagated linearly between connected banks. Here we relax this assumption and introduce a one-parameter family of non-linear propagation functions. As a case study, we apply this algorithm to a data-set of 183 European banks, and we study how the stability of the system depends on the non-linearity parameter under different stress-test scenarios. We find that the system is characterized by a transition between a regime where small shocks can be amplified and a regime where shocks do not propagate, and that the overall the stability of the system increases between 2008 and 2013.

## I. INTRODUCTION

Complex networks [1–3] have proved useful to describe systems characterised by pair-wise interactions. Properties of dynamical processes on networks can be strongly affected by the underlying topology [4]. Examples include spread of news [5], rumours [6], diseases [7], financial distress [8], random walkers travelling the graph [9–11], and avalanches [12, 13].

In these cases stylized models, despite their apparent simplicity, can give meaningful indications on the large scale dynamic of the system [7], also helping to shed light on the importance of the network topology [14]. For example, models of epidemic contagion (such as SIS or SIR [15]) display dramatically different behaviors depending whether they take place on regular lattices or on complex networks. Similarly, also the spread of distress [16–19] in financial networks is deeply dependent on the pattern of connections among financial institutions. In particular, it is not possible to identify a single topology that can be considered robust with respect to all types of shocks [20, 21].

Financial institutions are strongly interconnected in a variety of ways (e.g. ownership relationships [22, 23], common asset holdings [24–26], trading of derivatives [27, 28], possible arbitrage opportunities to exploit [29]) through which distress can propagate and lead to amplification phenomena, such as default cascades. Here we focus on a single layer of interconnectedness, namely that associated with interbank loans. To cope with fluctuations of liquidity, banks constantly lend money to each other, at different maturities. Hence, lenders are subject to *counterparty risk*, i.e. the risk that their borrowers could default and therefore not be able to fulfill their obligations. This, in turn, could lead to the default of lenders, resulting in a further wave of distress.

In the literature on financial contagion, a bank is represented by its balance sheet, consisting of *assets* with a positive economic value (such as loans, derivatives, stocks, bonds, real estate) and of *liabilities* with a negative economic value (such as customers' deposits, debits). The *balance sheet identity* for bank  $i$  defines its *equity* as the difference between its total assets  $A_i$  and its total liabilities  $L_i$ :  $E_i = A_i - L_i$ . A bank with a negative equity would not be able to pay back its debtors, even assuming that it could sell all of its assets. Therefore, usually a negative equity is considered a good proxy for the default of a bank. An interbank loan extended by bank  $i$  to bank  $j$  is an asset for bank  $i$  and a liability for bank  $j$ . A natural way to represent such relationships is by means of a directed weighted network [30, 31] in which edges of weight  $A_{ij}^{IB}$  correspond to a loan of amount  $A_{ij}^{IB}$  from bank  $i$  to bank  $j$ . We call all other assets and liabilities *external* (see Fig. 1).

The study of the interbank network has attracted considerable attention, also for its practical importance. Two widely recognized algorithms to quantify losses due to financial contagion are the Furfin algorithm [12, 32] and

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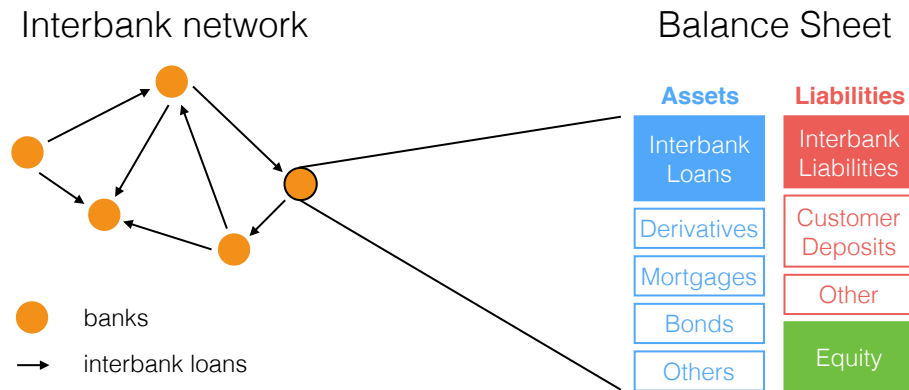


FIG. 1. Sketch of a portion of the interbank network and stylised representation of the balance sheet of a bank, with interbank assets and liabilities highlighted. The difference between assets and liabilities is the equity. A negative equity is usually considered a good proxy for the default of a bank.

DebtRank [33–39]. The former is essentially a threshold model according to which a bank propagates distress to its creditors only after its default. In contrast, DebtRank was introduced precisely to account for shock propagations occurring also in absence of default. To this end, relative losses in the equity of a borrower translate into the same relative devaluation of interbank assets of the corresponding lender. Those two mechanisms represent two extremes. On one hand, the Furfine algorithm is likely to underestimate the build-up of systemic risk. On the other hand, in DebtRank even tiny variations in the equity (as those deriving from daily market fluctuations) have a sizeable impact on the value of interbank assets. As a realistic scenario is likely to lie in-between those two extremes, in this paper we propose a model that interpolates between them and use it to perform stress tests to the European banking system. We will refer to the introduced model as non-linear DebtRank.

The paper is organized as follows: In Section II we specify the model used and present a detailed characterization of its behavior within the context of a case study. In Section III we discuss the main implications of our results, also from a policy-making perspective, and point out some limitations of our approach. We refer the reader interested in the details about the data used and a derivation of the algorithm to Section IV.

## II. RESULTS

We perform stress tests on  $N = 183$  publicly traded European banks using data from their balance sheets for the years from 2008 to 2013 (see Section IV for a detailed description of the data). Since data on bilateral exposures are not publicly available, we employ a reconstruction technique to infer plausible values [40, 41] and sample for each year 100 instances of interbank networks for given values of connectivity  $p$ , defined as the number of reconstructed edges divided by the number of possible edges ( $N(N - 1)$ ) (see Section IV for more details about data).

As in the linear DebtRank [39], a stress test consists in applying an initial shock to the system. The shock will then be propagated in time by an iterative map until convergence. From the point of view of risk management, the relevant quantity is the *relative equity loss* of bank  $i$  at time  $t$ :

$$h_i(t) = \frac{E_i(0) - E_i(t)}{E_i(0)}. \quad (1)$$

The corresponding quantity at the aggregate level is the *total relative equity loss*:

$$H(t) = \frac{\sum_{i=1}^N E_i(0) - E_i(t)}{\sum_{i=1}^N E_i(0)} = \sum_{i=1}^N \left( h_i(t) \frac{E_i(0)}{\sum_{j=1}^N E_j(0)} \right). \quad (2)$$

As we show in Section IV the dynamic equation for the relative equity loss is:

$$h_i(t+1) = \min \left\{ 1, h_i(t) + \sum_j \Lambda_{ij} [p_j^D(t) - p_j^D(t-1)] \right\}, \quad (3)$$

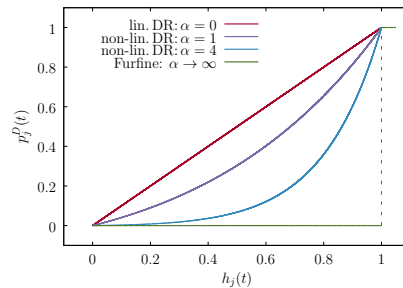


FIG. 2. Probability of default  $p_j^D(t)$  as a function of the relative equity loss  $h_j(t)$  for different algorithms. The non-linear DebtRank interpolates between the Furfine algorithm and the non-linear DebtRank.

where  $p^D(t)$  is the probability of default of bank  $j$  at time  $t$  and  $\Lambda_{ij} = A_{ij}^{IB}/E_i(0)$  is the interbank leverage matrix. In order to completely define the iterative map, we need to establish a relationship between  $p_j^D(t)$  and  $h_j(t)$ . In the case of Furfine’s algorithm, the probability of default is equal to one only if the equity is zero, and it is equal to zero otherwise, while in the linear DebtRank  $p_j^D(t) = h_j(t)$ . In the non-linear DebtRank we interpolate between these two cases using the function:

$$p_j^D(t) = h_j(t)e^{\alpha[h_j(t)-1]}, \quad (4)$$

where for  $\alpha = 0$  we recover the linear DebtRank, while for  $\alpha \rightarrow \infty$  we recover the Furfine algorithm. In Fig. 2 we show the probability of default as a function of the relative equity loss for Furfine, the linear DebtRank, and the non-linear DebtRank for two different values of  $\alpha$ . Such approach is purely “phenomenological”, in the sense that (4) is intended to provide an effective description of how stresses propagate and it has not been derived from accounting principles.  $\alpha$  should be therefore tuned so that the probabilities of default can be calibrated (in principle also diversely) for each bank. Intuitively,  $\alpha$  is related to the characteristic scale over which a variation in the relative equity loss has a negligible impact on the probability of the default. The importance of the interbank leverage matrix in the context of distress propagation has been already highlighted by [36, 39, 42], pointing out that the stability of the system is determined by its largest eigenvalue. Also in our case, using linear stability analysis, one can show that the stability of the system still depends on  $\Lambda$ .

Our stress test consists in assuming a devaluation of external assets by a factor  $x_{\text{shock}}$  for a fraction of banks equal to  $p_{\text{shock}}$ . All presented results are averaged both over the network samples and over the set of initially shocked banks (10 realizations of such set for each network in the sample).

A first analysis is focused on the most critical year: 2008. In Fig. 3 we show the effects of the initial shock, as its propagation through the network unravels in time. More in details, Fig. 3 shows  $S(t)$ , the fraction of stressed banks,  $D(t)$ , the fraction of defaulted banks, and  $H(t)$ , the total relative equity loss experienced by the system. Stressed banks are those which have experienced equity losses, but have not defaulted yet. Defaulted banks at time  $t$  are those for which  $h_i(t) = 1$ . From left to right, plots correspond to  $\alpha = 0$ ,  $\alpha = 1$ , and  $\alpha = 2$ .

The qualitative behaviour of both stressed and defaulted banks is shared by all panels. Stressed banks sharply increase in the first time steps and decrease afterwards, as defaults start to occur. This is consistent with the fact that stress propagates even in absence of defaults. However, a clear dependence from  $\alpha$  emerges. The most striking feature is that the time scale over which the system reaches its steady state is a non-monotonous function of  $\alpha$ . In fact, in both panels A and C convergence is reached before the first 20 time steps, while in panel B the dynamic is slower. This phenomenon can be intuitively understood as follows:  $\alpha$  is related to a soft threshold in the value of the relative equity loss that a bank needs to attain before it can propagate a shock. Such threshold is zero in the linear case and approaches one in the strongly non-linear regime ( $\alpha \gg 1$ ). In the first case shocks are quickly propagated through the network, while in the second case shocks are easily dampened. In the intermediate regime the build-up of the stress happens gradually. Nevertheless, the total relative equity loss can still reach values comparable to those of the linear case (see panels A and B).

Next, we present a comprehensive characterization of the steady state. Fig. 4 shows the surface plots of  $H_\infty$ , the total relative equity loss in the steady state, as a function of  $\alpha$  and  $x_{\text{shock}}$ , for network connectivity  $p = 0.05$ , and for different choices of  $p_{\text{shock}}$ . For a given value of  $p_{\text{shock}}$ , the range of  $x_{\text{shock}}$  spanned has been tuned so that the ranges of total shock  $p_{\text{shock}} \cdot x_{\text{shock}}$  affecting the system are equal across all the cases considered.

From the panel A of Fig. 4 we can clearly see that, for any value of  $x_{\text{shock}}$ ,  $H_\infty$  decreases monotonically with  $\alpha$ . In particular, it appears to be close to one up to a certain value of  $\alpha$ , after which it starts to decrease. At the same time, we observe that for large values of  $\alpha$ ,  $H_\infty$  decreases as  $x_{\text{shock}}$  becomes smaller. Finally, in the limit  $x_{\text{shock}} \rightarrow 0$

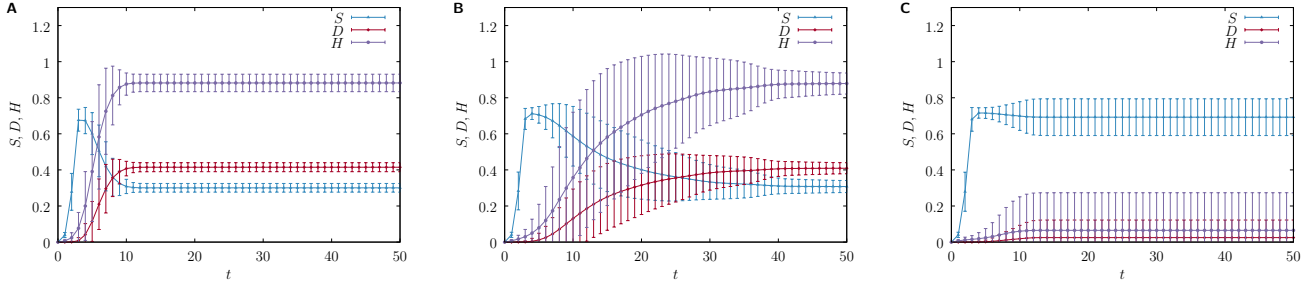


FIG. 3. Fraction of stressed banks  $S(t)$  (blue line), fraction of defaulted banks  $D(t)$  (red line), and  $H(t)$  (violet line), total relative equity loss experienced by the system as a function of the time  $t$  over which shock propagate. Banks experience a shock in the external assets, which suffer a relative loss equal to  $x_{\text{shock}} = 0.5\%$ . All points are averaged over a sample of 100 reconstructed networks with connectivity  $p = 0.05$  and compatible with 2008 balance sheets, and over 10 realisations of the shock in which each bank is shocked with probability  $p_{\text{shock}} = 0.05$ . Error bars span one standard deviation.  $\alpha = 0$  in panel A and the algorithm reduces to the linear DebtRank, while  $\alpha = 1$  in panel B, and  $\alpha = 2$  in panel C. We see that the dynamics unravels within a few time steps in the panels A and C, while it takes considerably more time steps in panel B.

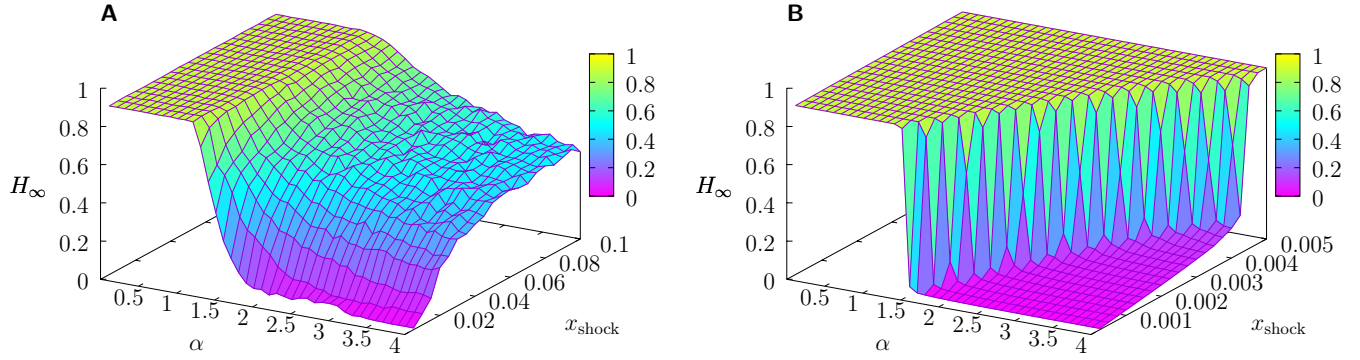


FIG. 4. Surface plot of  $H_{\infty}$ , the total relative equity loss in the steady state, as a function of the size of the shock suffered by external assets of banks  $x_{\text{shock}}$  and of the parameter  $\alpha$ , which tunes the non-linearity of the algorithm. All points are averaged over a sample of 100 reconstructed networks with connectivity  $p = 0.05$ , and compatible with 2008 balance sheets, and over 10 realization of the shock in which each bank is shocked with probability  $p_{\text{shock}} = 0.05$  (panel A) and  $p_{\text{shock}} = 1$  (panel B). Note that the range of the total size of the shock  $p_{\text{shock}} \cdot x_{\text{shock}}$  is the same for both panels. As  $\alpha$  increases, the propagation of the shock is dampened, resulting in smaller losses, and in two different regimes, whose separation is especially evident for  $p_{\text{shock}} = 1$ , i.e. when all banks are shocked.

the system displays a transition between a stable regime in which no losses occur, and an unstable regime in which also infinitesimal shock can lead to large total relative losses. Such transition can be easily seen in panel B of Fig. 4, where we present the results in the case in which all banks suffer the same initial shock. This can be interpreted as a shock to a risk factor common to all banks, such as a sudden change in interest rates or similar to that experienced during a major macroeconomic downturn. We note here that we have performed simulations for different values of the connectivity parameter  $p$  ranging between  $p = 0.05$  to  $p = 1$  (fully connected network). Interestingly, we observed that systems with very different connectivities behave in a similar way. A possible explanation is that, due to the reconstruction technique used (see Section IV),  $p = 0.05$  is already enough to connect systemically important banks.

Finally, in Fig. 5 we adopt the same setting as in Fig. 4 for different years. Overall, we observe that  $H_{\infty}$  markedly decreases from 2008 to 2013. It clearly emerges that the system was more prone to amplify shocks in 2008, when a region in parameters space in which  $H_{\infty} \simeq 1$  exists. This is consistent with the intuition that banks in 2008 were more fragile.

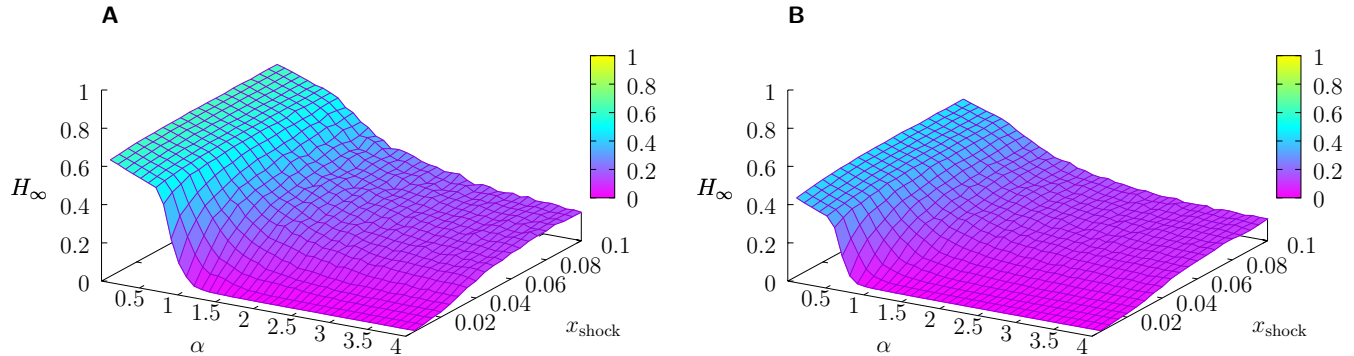


FIG. 5. Analogous of Fig. 4, but for the years 2010 and 2013. Here the connectivity of the reconstructed networks is  $p = 0.05$  and the fraction of shocked banks on average is  $p_{\text{shock}} = 0.05$ . Going from 2008 (see Fig. 4, panel A) to 2013  $H_{\infty}$  is less and less sensitive to changes in  $\alpha$  and it generally smaller, meaning that banks are more and more robust.

### III. DISCUSSION

In the present study, a general shock propagation mechanism is applied to an interbank network of 183 publicly traded European banks. With probability  $p_{\text{shock}}$  each bank is subject to an initial shock consisting in the devaluation of their external assets by a factor  $x_{\text{shock}}$ . The system reaction to shocks is measured in terms of the total relative equity loss, which takes into account the contribution of each bank to the relative equity loss of the system. The dynamic of shock propagation that we consider (non-linear DebtRank) interpolates through the parameter  $\alpha$  between two stress test algorithms: the Furfine algorithm and the linear DebtRank.

We notice that the propagation of shocks strongly depends on the parameter  $\alpha$ . In particular, in all stress scenarios that we have considered we observe a crossover between a regime of large losses (for small  $\alpha$ ), in which potentially all banks could default, and a regime of small losses (for large  $\alpha$ ), in which most banks survive the shock. The width of the intermediate region shrinks as the fraction of banks affected by the shock approaches one.

The model also shows that the interbank network was significantly more fragile in 2008, when the financial crisis took place, than in the subsequent years. This observation holds qualitatively for all values of model parameters and connectivities explored.

In addition to the properties of the steady state we have also looked into the dynamic of quantities such as the number of stressed and defaulted banks, whose behavior highlights the existence of different time scales, depending on the model parameters. For instance, we observe that the time needed to reach the steady state is a non-monotonic function of  $\alpha$ : in certain cases the shock produces a slow drive of the interbank network towards its collapse, while in other cases the crash occurs immediately after the shock.

Clearly, establishing a coherent mapping between probability of defaults and changes in equity opens several possible directions for future research. Obviously, calibrating  $\alpha$ , possibly extracting a different value for each bank, would represent a major achievement. Beyond the “phenomenological” approach adopted here, one could try to derive such relationship in the context of standard financial risk management theory. Moreover, here we have limited our analysis to direct exposures due interbank lending. A proper assessment of systemic risk should account for additional types of interconnectedness, such as that associated with overlapping portfolios, exchange of derivatives, and ownership structure. Hence, another future extension of the model could be based on a multilayer network that incorporates those effects. Complex interactions across different layers could lead to non-trivial amplification phenomena [43].

## IV. MATERIALS AND METHODS

### A. Model dynamics

In order to derive (3) we start from the balance sheet identity introduced in Section I in which we distinguish between external and interbank assets and liabilities:

$$E_i(t) = A_i^E(t) - L_i + \sum_{j=1}^N A_{ij}^{IB}(t) - L_{ij}^{IB}, \quad (5)$$

where assets depend explicitly on time, while liabilities do not. In fact, we can expect that the value of external assets fluctuates, while, as it will be clear in the following, the value of interbank assets  $A_{ij}^{IB}$  will be marked-to-market, depending on the probability of default of the borrower  $j$ . However, the fact that bank  $i$  reassess the value of its interbank claim towards bank  $j$  does not change the value of the debt that bank  $j$  owes to bank  $i$ : hence interbank liabilities (and analogously external liabilities) always keep their face value, and therefore do not depend on time. As a consequence, using (1) and (5) we can readily compute:

$$h_i(t) = \frac{A_i^E(0) - A_i^E(t)}{E_i(0)} + \sum_{j=1}^N \frac{A_{ij}^{IB}(0) - A_{ij}^{IB}(t)}{E_i(0)}. \quad (6)$$

In a scenario without recovery it is reasonable to assume that:

$$A_{ij}^{IB}(t+1) = A_{ij}^{IB}(0) (1 - p_j^D(t)), \quad (7)$$

which simply means that bank  $i$  updates the value of its interbank claims towards bank  $j$  such that it is equal to their face value if the probability of default  $p_j^D$  of the borrower bank  $j$  is zero and it decreases proportionally to  $p_j^D$  otherwise. By plugging (7) into (6) and using the definition of the interbank leverage matrix [36, 39]:

$$\Lambda_{ij} = \frac{A_{ij}^{IB}(0)}{E_i(0)} \quad (8)$$

we can immediately compute:

$$h_i(t+1) - h_i(t) = \frac{A_i^E(t) - A_i^E(t+1)}{E_i(0)} + \sum_{j=1}^N \Lambda_{ij} [p_j^D(t) - p_j^D(t-1)]. \quad (9)$$

In our stress test scenario we will initially shock external assets by a relative amount  $x_{\text{shock}}$ , i.e.  $A_i^E(1) = x_{\text{shock}} A_i^E(0)$ , such that  $h_i(1) = (1 - x_{\text{shock}}) (A_i^E(0)/E_i(0))$ . However, after the initial shock external assets do not change anymore and the evolution of the relative equity losses is entirely due to the re-assessment of the value interbank assets. As a consequence, the first term in the right-hand side of (9) is equal to zero, for  $t > 1$ . Finally, when the equity of bank  $i$  becomes equal to zero the bank defaults and is not able neither to further propagate shocks nor to sustain any additional losses. Hence, the maximum value attainable by the relative equity losses is one, which leads to (3).

From (3), we see that the results will depend on the relationship that we assume between the relative loss in equity of a bank and its probability of default. In the Furfine algorithm we have that:

$$p_j^D(t) = \begin{cases} 0 & \text{if } h_j(t) < 1 \\ 1 & \text{if } h_j(t) = 1, \end{cases} \quad (10)$$

while in the linear DebtRank:

$$p_j^D(t) = h_j(t). \quad (11)$$

In the non-linear DebtRank we interpolate between this two extreme cases by means of the parameter  $\alpha$  (see (4)): for  $\alpha = 0$  we recover the linear DebtRank, while for  $\alpha \rightarrow \infty$  we recover the Furfine algorithm.

## B. Data

The original source for raw data about balance sheets of banks is the Bureau van Dijk Bankscope database, from which we extract data for a subset of 183 among the largest European banks. In particular we use data about the total interbank assets (liabilities)  $A_i^{IB} = \sum_{j=1} A_{ij}^{IB}$  ( $L_i^{IB} = \sum_{j=1} L_{ij}^{IB}$ ) and we compute external assets (liabilities) as the difference between total assets (liabilities) and interbank assets (liabilities):  $A_i^E = A_i - A_i^{IB}$  ( $L_i^E = L_i - L_i^{IB}$ ). The same data have already been used in [36, 39]. See [36] for all the details about the handling of missing data.

As already pointed out, the balance sheet of bank  $i$  contains only data about its total interbank lending and borrowing, i.e. the values of  $A_i^{IB}$  and  $L_i^{IB}$ . As a consequence, the full matrix needs to be reconstructed by making some assumptions. Here we proceed as in [39] and postulate that the probability that bank  $i$  extended a loan to bank  $j$  is proportional to  $A_i^{IB}$ , the total amount of interbank lending of bank  $i$  and to  $L_i^{IB}$ , the total amount of interbank borrowing of bank  $j$ . The fitness model [44] allows us to compute the values of the probabilities  $\{p_{ij}\}$  that an edge  $i \rightarrow j$  exists for a given value of connectivity  $p$ . We then use  $\{p_{ij}\}$  to build a sample of 100 directed un-weighted networks for each year. For each network in the sample we assign weights using the iterative RAS algorithm [45]. See [39] for a full account of the procedure.

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## ACKNOWLEDGMENTS

All authors thank Marco D’Errico for sharing the data and for fruitful discussions and Stefano Battiston for the numerous exchanges of views. M. B. acknowledges support from FET Project SIMPOL nr. 610704. F. C. acknowledges support of the Economic and Social Research Council (ESRC) in funding the Systemic Risk Centre (ES/K002309/1). J. I. P., G. V., and G. C. acknowledge support from: FET IP Project MULTIPLEX nr. 317532, FET Project SIMPOL nr. 610704, FET project DOLFINS nr. 640772. G. C. acknowledges support from SoBIGData 654024 and CoEGSS nr. 676547. All authors have contributed to designing the experiment. J. I. P. and G. V. have run the numerical simulations. All authors have discussed the results and written the paper. The authors declare no competing financial interests. Raw data are available from Bureau van Dijk: <https://bankscope.bvdinfo.com>.