Comment on "First-Order Transition in the Breakdown of Disordered Media"

In a recent letter Zapperi *et al.* [1] investigate the critical behavior and the final breakdown suffered by a disordered fuse network when fed with increasing external current. They show that the final breakdown can be described as a first-order-like transition where the order parameter ϕ (the lattice conductance) displays a discontinuity in correspondence of a critical value of the driving parameter f (the external current). The work adds important contributions to the understanding of this kind of process. An interesting point that should be raised concerning the applicability of the mean field approach is the obtained critical value of the driving parameter at the breakdown. We point out that, while the mean field results of Ref. [1] predict a nonvanishing f_c, in the thermodynamical limit this critical value is 0.

The authors of Ref. [1] develop a mean field approximation scheme to map their fuse model into an exactly solvable model [2]. They show that the order parameter ϕ displays a critical behavior when the driving parameter approaches the critical value f_c: $(\phi - \phi_c) \simeq (f_c - f)^{1/2}$. The predicted behavior and the related mean field critical exponents are then checked by the authors by numerical simulations on a two dimensional random fuse networks and on a harmonic bond lattice subject to external stress. Finite values for both f_c and ϕ_c are found in agreement with mean field predictions.

On the other hand, investigations on several models for the breakdown of disordered media show that, for systems of finite size, f_c is a random quantity [3], the actual form of the probability distribution depending on the details of the model. In particular, it was also shown [4] by one of the authors of Ref. [1] that for some models with geometrical disorder f_c is distributed according to the Weibull's distribution, in general agreement with experimental observations. Depending on details of the model a somewhat different distribution [5] may occur. The common feature of these distributions is, however, that their maximum and their variance decrease with increasing system size and both vanish at infinite length scale. This means that the global failure process is ruled by extremal statistics [6]. Similar results, found also in some systems [7] with random rupture thresholds of bonds, imply eventually a trivial dynamics, with the whole phase diagram collapsed into the single point $f = 0$.

This is also the case for the class of models considered in Ref. [1]. Figure 1 shows the probability distribution of failure stress f_c for different sizes of a kind of vector Born model with random bond rupture thresholds [8]. We have simulated in this case the global failure process by driving the lattice with an increasing external stress which plays the role of the current. It can be seen that f_c shifts towards zero as the length scale increases, although the shift may become slower and slower at large sizes [5].

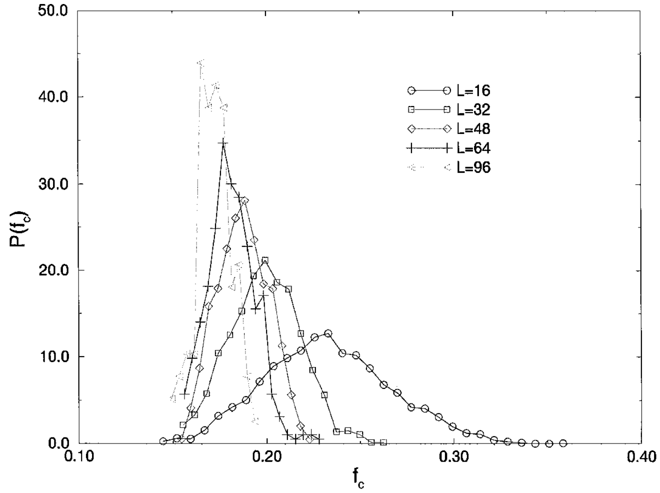

FIG. 1. Probability distribution for the breakdown stress shows to shrink to zero for increasing system size.

The finite value for f_c in Ref. [1] results from the fact that the fiber bundle model, on which the mean field maps the original systems, has actually a completely different geometry, displaying indeed a normal distribution of f_c with a nonvanishing expectation value in the limit of infinite size.

We acknowledge the support of EU Contract No. ERBFMRXCT980183.

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Received 7 October 1998
PACS numbers: 05.70.Ln, 62.20.Mk, 64.60.Fr

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