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Assessing financial distress dependencies in OTC markets: a new approach by Trade Repositories data.

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Abstract

After the recent financial crisis, it is undoubtedly recognized the importance of assessing not only the risk of distress for a single "financial entity", but also the distress dependencies between the different "entities", where by "entities" we mean in a broad sense any relevant cluster of products, risk factors, counterparties. In this paper, we focus on the Interest Rate Swap (IRS) segment as a significant fraction of the OTC market. We define a distress indicator by combining some distress drivers, such as averaged volumes, liquidity, volatility and bid-ask proxies. Hence, we analyse the distress dependencies among sub-markets identified by the segmentation of the IRS market according to contractual and financial features. We try to combine in an innovative way some new ingredients, namely the more granular data on OTC derivatives available from the trade repositories along with the classical JPoD approach introduced in the recent years by the IMF for studying the distress interdependence structure among financial institutions. The proposed technique seems to be quite promising. Indeed, the results are quite close to the practical intuition. At the best of our knowledge, this work is the first empirical study based on trade repositories' data for assessing systemic risk.

Keywords: financial distress interdependence, joint probability of distress, interest rate swap, systemic risk, trade repositories.

Jel codes: G01, G18, G19.

1 Introduction

The 2007-08 financial crisis was mostly due to liquidity and credit (counterparty) risks within the banking system. Even if the sources of risks were liquidity and OTC derivatives, one of the most surprising effects was the contagion to other financial players and to other markets and sectors. Consequently, this motivated the introduction of the *Systemic risk* as a new *building block* in the recent regulatory framework, such as the Basel III regulation (see e.g. BIS (2013) and FSB (2015)). In particular, the network theory approach was deeply applied in this field in order to find new useful measures to assess and to predict this risk. Financial networks have been widely exploited to describe the payments system, the interbanking deposit markets and the OTC derivatives markets. However, the latter is one of the most difficult to investigate due to the complexity of the "underlying" transactions, i.e.

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the derivatives' payoffs with their highly customized structures, and the scarce availability of detailed data especially in the past.

Aggregated statistics on OTC derivatives markets are usually provided by international organizations such as BIS (Bank for International Settlement) and OCC (US Office of Comptroller of the Currency), or banking associations like ISDA (International Securities and Derivatives Association). However, the collapse of 2007-08 stressed the need for a better provision of data in order for instance to assess systemic risk and prevent market abuse. Therefore, changes in regulatory frameworks have been pointed to a more detailed description of the deals, thus depicting a more representative and update picture of derivatives markets: some insights can be found e.g. in Russo (2010) and Duffie, Li, and Lubke (2010). In US, prior to the Dodd-Frank Act US 111th Congress (2010), financial institutions had less obligations regarding the amount of financial leverage, counterparty risk exposures, market share, and other data to be reported to any regulatory agency. Conversely, new rules introduce also requirements on OTC exposures and assign to specific agencies the role of collecting and sharing data. Similarly, in Europe the creation of the European Securities and Markets Authority (ESMA) and the European Systemic Risk Board (ESRB) are also motivated by the need to enforce the availability of data in order to improve the supervision and the restraint of the systemic risk (see EC (2013a) and EC (2013b)). In addition, the European Parliament established the European Market Infrastructure Regulation (EMIR) with the Regulation No. 648/2012 EUP (2012). Both the EMIR in Europe and the Dodd-Frank Act in US aim to disclose a more detailed description of the derivatives markets. Although only authorities are allowed to exploit the highest level of granularity, also market players can benefit from this flow of data through trade repository services (TRs), which collect and match data and allow the public access to this information[1]. In Europe this corresponds to an intermediate level where data are aggregated according to e.g. different asset classes and maturity features, while in US transaction data are reported almost in real-time and only confidential data are not available. As a deeper study of the divergences between EU and US in financial markets' regulation, we suggest e.g. Acharya et al. (2010), Lannoo (2013) and Valiante (2010). A good reference for a better understanding of the key requirements involved in the aggregation of TRs data are provided e.g. by the report FSB, Financial Stability Board (2014). Nevertheless, nowadays an increasingly need for transparency is required: for what it may concern over-the-counter derivatives and central counterparties we suggest Cecchetti, Gyntelberg, and Hollanders (2009) and Hull (2014), while for the interest rate derivatives markets some insights can be found in Avellaneda and Cont (2010) and Fleming et al. (2012).

Given the above context, in this paper we deal with the following issues. First, we identify a suitable set of sub-markets in the Interest Rate Swaps market. Of course, the identification of a robust sub-market concept is not an easy task. Along with some financial drivers that can support us for clustering the whole market, we must face some technical problems, such as the availability of a wide set of data and their related quality. Therefore, we limit our analysis on Fix-to-Floating instruments, taking into account contracts where the underlying rate is *USD-LIBOR-BBA*, contractual start is *Spot* and currency is *US Dollar*. This represents the most significant subset in our dataset (dataset which is supplied by IASON ltd[2]). The identification of sub-markets is then driven by the *maturity* of the contract, the *frequencies* of the swap legs and the presence of *clearing* agreements. Second, we define an indicator for assessing the level of distress present in these sub-markets. This distress indicator synthesizes several dimensions for market conditions, such as the bid-ask spread of the prices, the volatility of the prices, the number of deals and the averaged volumes of the traded deals. Third, we analyse the distress dependence between pairs of sub-markets (multidimensional generalizations of our analysis are possible without any technical problems). More precisely, by means of the copula theory, we investigate on the joint distribution of their distress indicator increments. In particular, we estimate Kendall's tau correlation coefficients and joint upper-tail probabilities (called joint probabilities of distress). Our approach is very similar to the one

1. For a detailed description of trade repositories activities, see e.g. DTCC (2013) and DTCC (2014).
2. For references see *http://www.iasonltd.com/*.

introduced in the IMF Banking Stability Measure report Segoviano and Goodhart (2009) to describe the distress interdependent structure among financial institutions. However, it is worthwhile to note that our context of application is completely different and, therefore, we need to face technical problems that are specific of our case-study (for instance, we do not have distress thresholds and so the CIMDO methodology used in Segoviano and Goodhart (2009) is not feasible in our case). For a review of fragility measures and models used for assessing systemic risk, we suggest Bisias et al. (2012) and Brunnermeier and Oehmke (2012).

Summing up, we can state that, although our approach belongs to the set of the standard methods used in risk management, at the best of our knowledge, the present work is one of the first empirical studies based on trade repositories' data. For instance, Slive, Witmer, and Woodman (2012) analyse central clearing effects in CDS markets through ICE Trust and ICE Clear Europe data, while Markose, Giansante, and Shaghaghi (2012) investigate the role of SIFIs within US CDS market using FDIC data. A very recent paper which exploits data from the Depository Trust and Clearing Corporation (DTCC) is Gehde-Trapp, Gündüz, and Nasev (2015); however it focuses on credit default swaps rather than on interest rate swaps. A comparison between official BIS statistics and detailed trade repositories data is in Bonollo et al. (2015) who describe how OTC derivatives market segmentation can be implemented trough the provision of the more granular flow of information due to the new regulatory framework. The novelties of our analysis are both the originality of the data-set that we exploited to identify specific sub-markets and the proposed distress indicator. We note that, despite the several difficulties to be faced due to the pioneering nature of our work (quality of the TRs data, new sub-markets identification, new distress indicator definition), our outcomes are consistent with the practical intuition. While analysing in a micro-prudential approach the portfolio and the risks of a bank is a complex but rather sharp task, to infer from global market data the risks of a financial system as a whole portfolio is a current frontier of the research. In the past, lack of detailed data and the difficulty to converge towards an accepted definition of systemic risk (see e.g. IMF BIS FSB, International Settlement, and Board (2009), FSB (2010) and Bonollo et al. (2014)) made very challenging the measurement of distress signals for the financial markets. This work aims to exploit in a useful way the trade repositories data to help to detect distress and crisis phenomena.

The paper is organized as follows: after a detailed description of the dataset and of the procedure applied for sub-markets identification (Section 2), we propose an indicator to investigate distress dependencies among our sub-markets (Section 3). Then, Section 4 for explaining in detail our methodology follows. Finally, the results of our analysis are illustrated and discussed in Section 5 and Section 6 closes the paper with some remarks and future lines of research.

2 Description of the dataset

Until 2013 the international statistics about this market were provided by some organizations such as BIS and OCC, or banking associations like ISDA. These statistics are based on some main *reporting dealers*, e.g. the biggest (a few dozens) commercial and investment banks that send, with a given frequency, some low granularity data about their own derivatives deals to central organizations which publish them after having applied data cleansing to avoid e.g. double counting issues. Although the panel for data covers a high percentage of the OTC world markets, the information related to the asset classes and to the payoffs is not very rich. Moreover, since this reporting is not "mandatory" in a strict sense, then one could doubt about the data quality of the process. In fact, the statistics released by different organizations are not exactly comparable, i.e. the aggregated metrics (e.g. the notional or the mark to market) do not re-conciliate themselves in a very satisfactory way. For these reasons, we rely on the dataset provided by *GTRAnalytics*[3], a software developed by the

3. For references see *http://www.financial-machineries.com/gtr-analytics.htm*.

consulting firm IASON ltd, which collects trades' information from several trade repositories and for many types of contracts, controlling for manifest inconsistencies and mismatches. This process limits potential biases due to data misreporting and fragmentation which arise from merging datasets from different trade repositories and among different regulations.

Our study focuses on the interest rates derivatives market which at the end of December 2014 accounts for the 80% and 75% of the global OTC derivatives market in terms of the notional amount outstanding and the gross market value, respectively. In particular, since the swaps market was worth $381 trillion compared with $505 of the total outstanding notional amount of the interest rate market[4], this motivates our choice to study the swaps segments as a representative case-study for the global OTC derivatives market. For each deal (identified by an ID) our database specifies the asset class of the instrument and reports a set of information regarding contractual terms, including: the execution time, the effective date and the contract expiry of the deal, the settlement and both the underlying assets currencies, payment frequencies, day count convention, and, obviously, the notional and the price. In addition, we can also exploit information on clearing agreements and collateral positions which enrich the description of market trends and improve risk assessment. As a matter of fact, it is worthwhile to outline that we refer to prices and volumes of actual traded deals in the market, which consequently extend the traditional use of *offered* rates (bid/ask quotes shown by brokers or data providers) and *consensus* (quotes/prices submitted by market contributors) data.

2.1 Sub-markets identification

The identification of a robust sub-market concept is not an easy task. Along with some financial drivers that can support us for clustering the whole market, we must take into account some technical issues, such as the availability of a wide set of data and their related quality. Hence, although the methodology we propose is still a bit heuristic, we believe that at this first stage of the research it is a reasonable approach.

We apply the following procedure. In order to ensure comparability, we limit our analysis on *Fix-to-Floating* instruments. For the same reason, we consider contracts where the underlying rate is *USD-LIBOR-BBA*, contractual start is *Spot* and currency is *US Dollar*. This represents the most significant subset in our dataset. In particular, the identification of sub-markets is driven by the *maturity* of the contract, the *frequencies* of the swap legs and the presence of *clearing* agreements. Data investigation suggests to consider Fix-to-Floating instruments with leg frequencies equal to *(3m vs 3m)* and *(6m vs 3m)*. In addition, we aggregate deals according to three main maturities, i.e. less or equal to 2 years (*Short*), between 2 years and 10 years (*Medium*) and greater or equal to 10 years (*Long*). Finally, we distinguish contracts between those for which there are clearing agreements (C) from those for which uncleared (UC) conditions are present.

Let us point out some facts about these sub-markets. The frequency of the leg payments became a relevant factor after the financial crisis, when it was clear that the frequency of cash flows changed both the liquidity (funding) risk and the counterparty risk for the two involved financial agents. This is well known as the *multiple curve new framework* [5]. In other words, one cannot evaluate financial instruments without considering the frequency of cash flows since *ceteris paribus* the IRS fair values will be slightly different. Also the netting flag is a very informative variable. For instance, both the Dodd-Frank act and the ESMA regulation ask to the banks to apply in a mandatory way the netting agreement in the transaction management in order to keep as low as possible the credit exposures. In addition, enterprises are obliged to this practice only above some relevant thresholds (e.g.

4. Data refer to BIS statistics and to single currency contracts only. For further references, see *http://www.bis.org/statistics/derstats.htm*.
5. See Pallavicini and Brigo (2013).

Table 1: Sub-markets Definition

Sub-mkt	**Fix-to-Floating**	**Maturity**	**Clearing**
1	*(3m vs 3m)*	*Short*	*C*
2	*(3m vs 3m)*	*Medium*	*C*
3	*(3m vs 3m)*	*Long*	*C*
4	*(6m vs 3m)*	*Short*	*C*
5	*(6m vs 3m)*	*Medium*	*C*
6	*(6m vs 3m)*	*Long*	*C*
7	*(6m vs 3m)*	*Short*	*UC*
8	*(6m vs 3m)*	*Medium*	*UC*
9	*(6m vs 3m)*	*Long*	*UC*

Fix-to-Floating refers to contracts with swaps legs frequencies equal to *(3m vs 3m)* or *(6m vs 3m)*. *Short*, *Medium* and *Long* refer to deals with maturities less or equal to 2 years (*Short*), between 2 years and 10 years (*Medium*) and greater or equal to 10 years (*Long*). Finally, data are further partitioned according to the presence (*C*) or absence (*UC*) of clearing agreements.

3 bn of Euro of outstanding notional for interest rate derivatives in the ESMA regulation). For this reason, we might assume that the Yes/Not clearing agreement digit can be used as a proxy for the counterparty class, i.e. banks vs. enterprises.

Although information on traded deals is available also for the first part of 2013, for the following analysis we consider only data from *September 2013* to *April 2015* since the amount of reported deals at the beginning of 2013 is not satisfactory. This choice ensures a good availability of data along the reference period. The following tables show the descriptive statistics for each sub-market[6].

Table 2: Number of deals and their notional amounts (in million of Us Dollar)

IRS (3m x 3m)	*Short*		*Medium*		*Long*		***Total***	
	Cleared	*Uncleared*	*Cleared*	*Uncleared*	*Cleared*	*Uncleared*	***Cleared***	***Uncleared***
Number of deals	1,170	154	9,028	706	9,406	133	**19,604**	**993**
Notional amount	175,226	8,359	874,483	34,479	412,779	4,131	**1,462,488**	**46,969**

IRS (6m x 3m)	*Short*		*Medium*		*Long*		***Total***	
	Cleared	*Uncleared*	*Cleared*	*Uncleared*	*Cleared*	*Uncleared*	***Cleared***	***Uncleared***
Number of deals	14,063	2,128	100,555	9,547	92,008	9,405	**206,626**	**21,080**
Notional amount	2,301,060	294,994	9,258,744	745,467	4,134,122	396,999	**15,693,926**	**1,437,460**

Descriptive statistics refer to the number of deals and their notional amounts (in million of Us Dollar) from September 2013 to April 2015. In the upper part we show data for contracts with swaps legs frequencies equal to (6m vs 3m), while in the lower part we consider deals with swaps legs frequencies equal to (3m vs 3m). *Short*, *Medium* and *Long* refer to deals with maturities less or equal to 2 years (*Short*), between 2 years and 10 years (*Medium*) and greater or equal to 10 years (*Long*). Finally, data are further partitioned according to the presence of clearing agreements.

Results suggest that deals involving *Fix-to-Floating* instruments with leg frequencies equal to (6m vs 3m) are more frequent than those with leg frequencies equal to (3m vs 3m). This is more evident once we consider deals characterised by clearing agreements. In particular, short maturities are less diffused, while figures are comparable for the *Medium* and the *Long* sub-sets. In order to identify sub-markets, these descriptive statistics suggest to discard uncleared deals for *Fix-to-Floating* instruments with leg frequencies equal to (3m vs 3m) due to data limitations. Therefore, our final list of sub-markets is composed by six sub-sets with leg frequencies equal to (6m vs 3m) and three sub-sets with leg frequencies equal to (3m vs 3m), the latter characterised by the presence of clearing agreements.

6. We apply a further check for double counting in the transactions by controlling for contractual terms. In particular, we consider as duplicated deals those transactions that present the same dissemination id, contract expiry, effective and end dates, prices and notional amounts.

Official BIS descriptive statistics provide information for OTC derivatives by currency. At end-December 2014[7] interest rate swaps were 124 trillion of US Dollar in terms of notional amount outstanding, while in our dataset (*Fix-to-Floating* 3m3m plus 6m3m) the amount is about 14.6 trillion of US Dollar, i.e. close to the 12% of the whole USD IRS market. Although a direct comparison between BIS statistics and our sample would require a more detailed partition of the deals, not yet available in the BIS statistics, we still observe a satisfactory coverage of IRS markets provided by our dataset. Hence, we can roughly guess that our dataset covers at least 20% of the comparable worldwide IRS market of the *Fix-to-Floating USD-LIBOR-BBA* with leg frequencies *3m-3m* and *6m-3m*.

3 Distress indicator

In this work we attempt to assess the level of distress present in a sub-market. Given a certain set of sub-markets representative for the global OTC market of swap instruments, we propose a way to measure their market conditions and to identify whether pairs of sub-markets are reciprocally co-dependent. In particular, we are interested in sub-markets' co-movements that point to distressed scenarios. Therefore, we need to introduce an indicator of distress which synthesizes several dimensions of market conditions. Before exposing how this measure is defined, it is worthwhile to stress that we do not rely on traditional concepts of default since markets cannot go bankrupt in a strict sense, although the absence of transactions can be interpreted in a similar way.

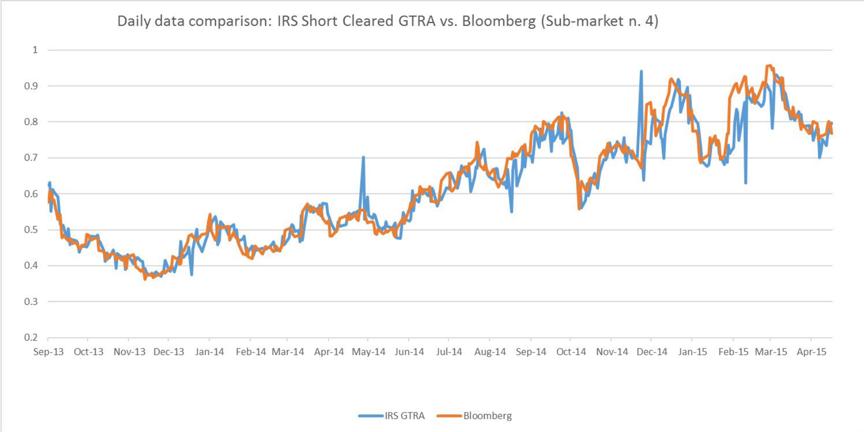
On the other hand, we recall that even in the Basel Vasiceck-Gordy model, although single default probabilities are present, the choice e.g. of the 99.9% quantile for the capital charge is not related to a specific event, since it is just a very regulatory confidence level for the estimation of the global credit portfolio's losses. Therefore, it seems reasonable to avoid the selection of a given threshold above which we state that the sub-market experiences distress. Thus, we suggest to analyse sub-markets' reciprocal behaviours in the tail corresponding to detrimental conditions. Obviously, a threshold level could be set up in some further research to design e.g. a proper *backtesting* procedure for the model. Finally, before giving the technical details, let us recall that the IRS price level represents an average of the *forward* (expected) interest rates over the IRS maturity. Hence, any turmoil in the IRS price and/or observed volumes could jointly reflect market, counterparty and liquidity aspects.

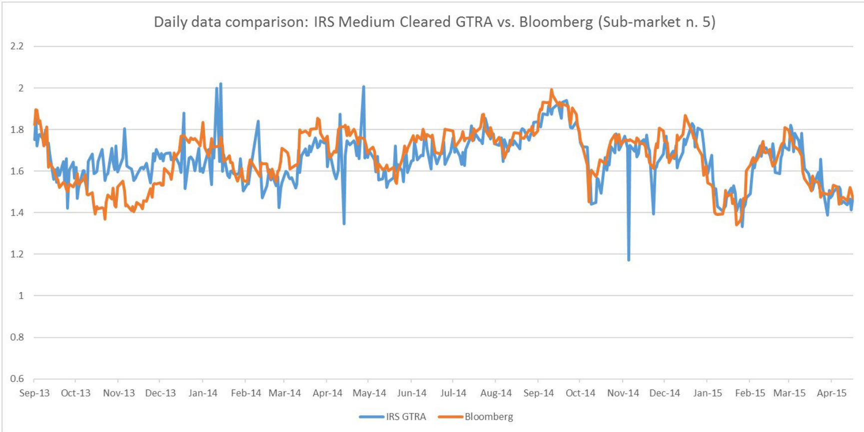
In order to assess the level of financial distress within each sub-market, we propose an indicator of distress to capture some aspects related to the financial stability. We assume that the main forces affecting the level of distress of a sub-market are represented by *i)* the bid-ask spread of the prices, *ii)* the volatility of the prices, *iii)* the number of deals and *iv)* the averaged volumes of the traded amount. These hypotheses reflect the perception that a wider bid-ask spread stands for deteriorated liquidity conditions as well as a higher volatility of prices may suggest the presence of a distressed scenario. Similarly, a lower number of deals (or modest average notional amounts) may be a signal of slowness in the process of adjusting prices, which may impact on the capacity to close positions. In addition, these forces may present interacting effects. In our work, we prefer to rely on a simple approach, thus focusing only on the direct contributions of the four aforementioned measures.

For the point *i)*, since we cannot directly deal with bid-ask quotes and we are not aware of the parts involved in the transactions, we rely on the ratio between the maximum and the minimum of daily prices as an acceptable proxy for the bid-ask spread within a certain sub-market. This choice is motivated by the fact that a tight daily deviation between the maximum and the minimum is likely to imply that traded deals have been priced within a close interval. For point *ii)* we compute the dispersion in terms of the standard deviation of

7. For references, see *http://www.bis.org/statistics/dt07.pdf.*

Figure 1: Comparison between GTRA and Bloomberg data



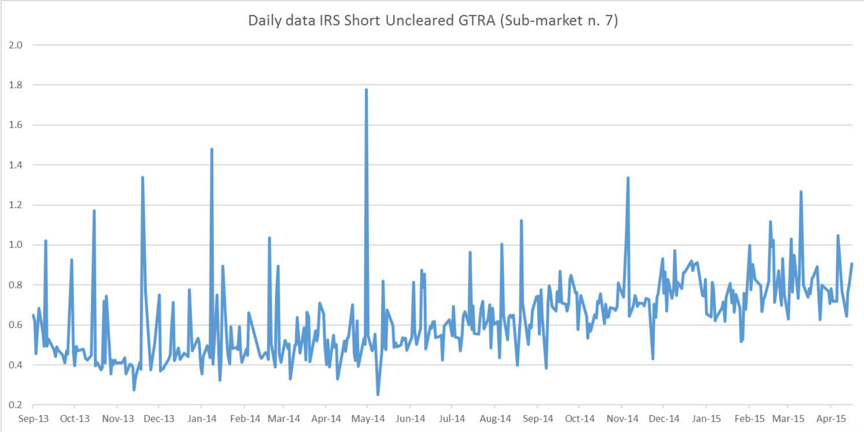


daily prices, while for points *iii)* and *iv)* we determine the daily number of deals (cardinality) and the daily average of the traded notional amounts, respectively. Finally, in order to get less noised estimates we aggregate these measures on a weekly interval [8].

Before analysing the characteristics of the distress indicators, let us discuss some more basic statistics related to the quality of our dataset. In *Figure 1* (top in the panel), we compare the aggregate IRS prices in GTRA for the short maturity bucket in the cleared case (sub-market n. 4) vs the data we obtain from *Reuters* corresponding to the USD 2Y curve. Time series appear very similar during the entire reference period with only few exceptions. The same analysis for the medium maturity bucket (sub-market n. 5) is shown in the bottom of the panel (where we compare vs USD 5Y curve). In this case we can observe some differences especially in the first period, although on average both sources of data depict a similar picture. This might be due to a grouping effect, since the medium bucket is mainly affected by 5Y tenor, but also some other maturities (e.g. 3Y, 4Y, 7Y) may influence the aggregated level. Finally, in *Figure 2* we plot the time series for the un-cleared case with short maturity (sub-market n. 7). We can observe a quite irregular dynamics. We recall that the "un-cleared" flag represents a signal for the fact that the counterparty is likely to be a corporate instead of another bank. In these situations different factors could imply an apparently strange behaviour of the price series:

8. If missing values are present due to lack of data, we replace missing values by the cubic spline interpolation of the available points.

Figure 2: GTRA Price time series for sub-market n. 7



- an *up-front* is stated in the deal, i.e. one of the two counterparties receives immediately a cash amount. To balance it, the IRS fixed leg might be shifted to offset the upfront.
- the IRS pay-off could be very customized, hence it requires a different fixed level.
- there is less liquidity in the IRS segment for the enterprises, and banks apply some relevant *mark-up* to off-set the counterparty risk or to get a positive profit.
- a combination of the above factors.

With regards to the distress indicator, the following tables provide a summary description of the single components involved in the definition of the distress indicator. In particular, for each sub-market we show the average (monthly or quarterly) of the daily observations of respectively: the (ln) of the ratio between the maximum and the minimum, the dispersion, the number of deals and the average notional traded amount. These statistics allow us to depict how sub-markets have evolved over time, thus suggesting the emergence of some common pattern that might have affected the overall behaviour as well as the presence of specific features that characterise certain sub-markets.

Table 3: Mean period (ln) deviation between the maximum and the minimum

Sub-mkt	SEP 2013	IV Q 2013	I Q 2014	II Q 2014	III Q 2014	IV Q 2014	I Q 2015	APR 2015
1	0.35	0.14	0.10	0.15	0.37	0.37	0.49	0.37
2	0.29	0.24	0.19	0.58	0.71	0.65	0.54	0.55
3	0.05	0.04	0.04	0.19	0.23	0.23	0.20	0.27
4	0.55	0.42	0.54	0.48	0.75	0.86	0.81	0.64
5	1.06	1.06	1.08	1.00	0.83	0.81	0.70	0.76
6	0.43	0.41	0.33	0.34	0.30	0.31	0.42	0.36
7	0.54	0.51	0.80	0.57	0.72	0.66	0.79	0.30
8	1.10	0.98	1.09	0.99	0.83	0.79	0.68	0.62
9	0.53	0.44	0.39	0.33	0.36	0.34	0.33	0.40

Table 4: Mean period dispersion (standard deviation)

Sub-mkt	SEP 2013	IV Q 2013	I Q 2014	II Q 2014	III Q 2014	IV Q 2014	I Q 2015	APR 2015
1	0.34	0.11	0.07	0.04	0.11	0.10	0.15	0.12
2	0.29	0.26	0.24	0.30	0.32	0.29	0.21	0.21
3	0.20	0.10	0.09	0.25	0.28	0.25	0.17	0.20
4	0.06	0.04	0.04	0.05	0.09	0.09	0.11	0.08
5	0.43	0.40	0.46	0.43	0.36	0.32	0.25	0.24
6	0.33	0.37	0.34	0.32	0.28	0.25	0.19	0.19
7	0.15	0.16	0.20	0.13	0.17	0.17	0.22	0.10
8	0.53	0.49	0.51	0.48	0.41	0.36	0.31	0.28
9	0.44	0.41	0.37	0.34	0.31	0.27	0.24	0.34

Table 5: Mean period cardinality (number of traded deals)

Sub-mkt	SEP 2013	IV Q 2013	I Q 2014	II Q 2014	III Q 2014	IV Q 2014	I Q 2015	APR 2015
1	2.29	1.73	1.48	4.20	5.74	7.45	5.13	4.35
2	3.09	3.02	2.66	24.23	51.01	46.22	35.94	31.58
3	1.80	1.96	2.64	25.29	46.41	47.21	46.97	37.61
4	24.71	28.55	39.15	33.18	40.83	57.19	50.27	48.36
5	183.16	252.00	292.24	257.18	294.99	332.27	309.83	244.12
6	195.48	241.54	228.13	223.52	272.65	311.14	320.49	251.65
7	6.41	5.25	6.34	5.17	7.24	9.59	8.47	4.48
8	25.41	21.04	25.61	24.43	33.17	40.68	22.50	16.50
9	30.15	25.04	21.62	18.39	35.75	44.10	21.27	13.60

Table 6: Mean period average volumes (notional traded amounts in million of US Dollar)

Sub-mkt	SEP 2013	IV Q 2013	I Q 2014	II Q 2014	III Q 2014	IV Q 2014	I Q 2015	APR 2015
1	20.00	65.10	62.70	129.40	164.19	167.57	175.55	169.81
2	30.38	32.43	43.83	76.00	98.28	102.73	98.68	104.40
3	30.88	31.95	39.39	37.31	41.92	46.80	44.95	40.85
4	153.77	166.49	152.82	157.93	144.77	161.41	186.18	178.76
5	77.18	95.51	94.38	93.21	91.00	93.86	89.74	87.40
6	43.00	49.11	46.08	45.13	43.53	45.36	42.82	42.54
7	84.48	142.72	138.34	148.59	136.43	129.29	153.99	205.69
8	55.93	67.25	83.19	78.58	72.23	81.10	83.98	98.13
9	36.21	40.05	50.21	42.99	38.39	41.08	43.28	46.95

Descriptive statistics provide some insights on sub-markets' behaviours during the sample period. As regarding the (ln) max/min deviations, for some sub-markets (1, 2, 3, 4) the first part of 2014 coincides with low mean values, while in the recent period they reach wider deviations. Conversely, other sub-markets (5, 6, 8, 9) show flattening or even declining trends during the reference period. These patterns are on average confirmed when we consider the estimates for dispersions. In addition, one might be interested in disentangle whether sub-markets with common contractual terms share similar trends. For instance, the absence of clearing agreements (sub-markets 7, 8, 9) seems to do not greatly affect the overall picture, since pairs of sub-markets (e.g. 5-8 and 6-9) with the same maturity and the same swap frequencies legs but different clearing agreements exhibit closer estimates. In addition, as we expect for sub-markets with high volumes of transactions, those with cleared conditions (from 4 to 6) show a smaller price dispersion than the respective un-cleared sub-markets (from 7 to 9). Finally, even for sub-markets with different swap leg frequencies but with the same maturity, we can observe that the price dispersion are quite close in some cases, e.g. the cluster 2 vs. the parallel cluster 5. Moreover, it may be the case that a sub-market has a high max-min deviation but low dispersion (e.g. sub-market 4), thus supporting the use of a set of measure in order to disentangle the overall level of distress for a certain sub-market. Finally, the last two measures point out a picture characterised by increasing trends in both the number of deals and the average notional traded amounts, although estimates for the last period seem to indicate a renewed decrease of the transactions. In some cases (e.g. sub-markets 5-6) although the average cardinalities are similar, the average notional traded amounts are quite different. Therefore, the overall picture provided by these estimates suggest that a reasonable indicator of sub-market's conditions should rely on a comprehensive set of measures able to capture several market dimensions. For these reasons, we propose the following indicator of distress:

$$I_{i,t} = \ln\left(\frac{max_{i,t}}{min_{i,t}}\right) \times \frac{\sigma_{i,t}}{(Avgvolume_{i,t} \times Num_{i,t})} \tag{1}$$

where $i = 1, ..., S$ and $t = 0, ..., T-1$ are the indexes for the sub-markets and the weekly observations, respectively. Symbols max and min denote the maximum and the minimum of the weekly prices for each sub-market i at time t, respectively. Quantity $\ln\left(\frac{max_{i,t}}{min_{i,t}}\right)$ is lower-bounded and increases when the deviation between the max and the min becomes larger. The symbol σ stands for the standard deviation of the prices: its impact on the indicator of distress is positive, as greater volatility might be associated with distressed market conditions. Conversely, Num (i.e. the number of deals) has a negative effect since it is assumed that more traded deals imply that it is easier to find a counterparty, thus limiting the risk of liquidity risk. Lastly, the use of mean volumes ($Avgvolume$) indicates the average notional traded value of the deals and it is introduced for liquidity purposes. Therefore, we decide to consider explicitly each driver (i.e. the deviation max/min, the dispersion, the cardinality and the average traded amount) in the formula for the sake of clarity, although we are aware that there are some redundant issues related to the use of Num in the estimates of both dispersion and average volumes. In addition, one could argue that formula (1) can be improved by generalizing it with some parameters to be calibrated in some optimal way, such as:

$$I_{i,t}(\alpha, \beta, \gamma, \delta) = \ln\left(\frac{max_{i,t}}{min_{i,t}}\right)^{\alpha} \times \frac{\sigma_{i,}^{\beta}}{(Avgvolume_{i,t}^{\gamma} \times Num_{i,t}^{\delta})} \tag{2}$$

However, at this step of the research, we believe that is better to work with a simpler indicator that captures in a qualitative way (increasing or decreasing) the impacts of the different distress factors, thus focusing on the first empirical results. Below, we show the average (monthly or quarterly) of the weekly observations of the indicator of distress as defined above.

Table 7: Mean period indicator of distress (values multiplied by 10^9)

Sub-mkt	SEP 2013	IV Q 2013	I Q 2014	II Q 2014	III Q 2014	IV Q 2014	I Q 2015	APR 2015
1	12.21	1.63	0.44	0.24	0.16	0.11	0.17	0.10
2	9.61	1.97	1.88	0.28	0.05	0.05	0.12	0.04
3	0.09	0.32	0.05	0.28	0.04	0.03	0.02	0.04
4	0.01	0.01	0.01	0.01	0.03	0.01	0.01	0.01
5	0.04	0.03	0.03	0.02	0.01	0.01	0.01	0.01
6	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01
7	0.29	0.51	0.42	0.22	0.31	0.22	0.30	0.05
8	0.46	0.62	0.32	0.34	0.18	0.13	0.16	0.15
9	0.24	0.23	0.15	0.25	0.11	0.06	0.12	0.28

Table 7 shows how sub-markets' distresses have evolved over time. These estimates reflect the joint contributions of the single measures introduced above. In order to assess the level of distress within a sub-market one should in principle observe the magnitude of this measure, since by construction higher values correspond to deteriorated market condition under our assumptions. With regard to *Table 7*, we observe some stylized relevant facts[9]. The distress indicator by its construction does not have a practical or physical meaning although it allows some qualitative insights by looking at the *ranking* between the different markets. Hence it is worthwhile to note that the sub-markets from 4 to 6 (which involve *bank-to-bank* most liquid sub-markets) show a very low distress level. If we analyse the other sub-markets (from 7 to 9), it seems that the un-cleared ones (usually deals between *bank-to-enterprise*) exhibit more risky figures. This is mainly due to the lack of liquidity and or large min-max range.

In other words the indicator allows us to capture in a formal and intuitive way the causal forces that could move the sub-markets towards a distressed state. In order to switch from an useful but still descriptive study to the investigation of how sub-markets are reciprocal influenced, we analyse how pairs of sub-markets are jointly dependent, i.e. how sub-markets' distresses co-move. Therefore, we attempt to study the dependence in the movements of this indicator. For each sub-market, we compute the following increments:

$$X_{i,t+1} = \frac{I_{i,t+1} - I_{i,t}}{I_{i,t}} \tag{3}$$

for $i = 1, \dots, S$ and $t = 0, \dots, T-1$. Hence, a sub-market that exhibits positive increments implies that it experiences deteriorated conditions which became more serious if these variations become larger. For these reasons, our analysis is focused more on the right tail of the distributions of the increments, which corresponds to distressed market conditions.

9. Estimates for September 2013 might even reflect the *backload* process of the deals. For instance, in EU the EMIR regulation was practically applied from February, 2014. At that time also the deals already alive were uploaded by a massive *backload* process. Hence we can doubt about the quality of the oldest data. In fact from the effective trade repository feed running process the distress indicators become lower and more stable. As a further remark, let us note that the *VIX* popular index, i.e. the volatility index of the S&P index level, did not touch at the end of 2014 any abnormal levels. In September 2013 the average level was 14.65%, just 50 bps higher than the average level of 2014, 14.14%.

4 Methodology

As already said, the "distress" is an extreme event, which can be seen as an upper tail event related to the process that describes the movement of the sub-market's status. Therefore, we aim at providing, for each pair of sub-markets, a *Joint Probability of Distress*[10], that is the joint probability that both sub-markets simultaneously exhibit increments of the distress indicator above some threshold. This approach is similar to the one in Segoviano and Goodhart (2009), where the indicators known as Banking Stability Measures are presented. Our methodology is similar in the facts of viewing markets' players as a portfolio of players, and providing a distress interdependence structure, which is able to capture not only linear correlation but also nonlinear distress dependencies among the players in the system.

In order to compute the joint probabilities of distress, we split the analysis into three parts. First, once sub-markets have been set up, we study the form of correlation between each couple of sub-markets and how strong this relationship is. We exploit the family of Archimedean bivariate copulas (more precisely: *Clayton*, *Gumbel* and *Frank* copulas). The general theory of copulas states that a joint distribution of some random variables can be decomposed into a function (called copula) that describe the interdependence structure among the considered variables and their marginal distributions (see the Appendix for more technical details). The reason to narrow the choice of copula functions among the family of the Archimedean ones lie in the fact that we want the possible dependencies to be comparable. In addition, the Archimedean family provides, through a unique parameter (i.e. θ), a proxy for the dependence degree between the two sub-markets. After having identified the dependence structures for each pair of sub-markets, we produce a ranking based on the Kendall's tau correlation coefficients. Finally, we compute joint probabilities of distress at different marginal threshold levels.

Note that in this work we limit ourselves to the bivariate copulas in order to study the dependence for pairs of sub-markets, but multidimensional generalizations are obviously possible.

In the following three subsections, we illustrate the technical details of the three steps of our analysis: the identification of the copula function for each possible pair of sub-markets, the global ranking classification (based on Kendall's tau) among different pairs of sub-markets, and the computation of the JPoD for couples of sub-markets. According to the latter probabilities a final ranking classification of the pairs of sub-markets is provided, too.

4.1 The preliminary copula-based procedure

Given S sub-markets and, for each sub-market, T time-observations of the random variable of interest X (described above by formula (3)), we can represent the data by means of a real-valued matrix X of dimension $S \times T$,

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1t} & \cdots & x_{1T} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{it} & \cdots & x_{iT} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{S1} & \cdots & x_{St} & \cdots & x_{ST} \end{bmatrix} = \begin{bmatrix} x_{1\cdot} \\ \vdots \\ x_{i\cdot} \\ \vdots \\ x_{S\cdot} \end{bmatrix}$$

where x_{it} represents the value of the observation t for the sub-market i and $x_{i\cdot}$ is the row-vector that contains all the values related to the sub-markets i.

10. The meaning of joint distress of couples of sub-markets, as well as the related terminology, are referred to concepts introduced in this paragraph and in Section 3. Hereinafter, any reference to existing expressions has to be considered contextualized to our work.

The preliminary procedure we propose takes as input this matrix and returns for each couple of sub-markets the most appropriate Archimedean copula and the corresponding parameter θ:

1. first of all the procedure derives the margin for each sub-market i by finding the empirical cumulative distribution function $\widehat{F}_i$ based on the corresponding T-dimensional row $x_{i\cdot}$. For each actor i, we are assuming the values $x_{i1}, \dots, x_{iT}$ as i.i.d. realizations drawn from the same univariate distribution.

2. Fixed a pair of different sub-markets, say (i, j), for each copula type (Cl =Clayton, Gu =Gumbel, Fr =Frank), the procedure computes the maximum value of the copula loglikelihood and the corresponding estimated value of the dependence parameter. Formally, it maximizes the function defined as

$$\theta \mapsto \ell_{(i,j),type}(\theta) = \sum_{t=1}^{T} \ln c_{type}\left(\widehat{F}_i(x_{it}), \widehat{F}_j(x_{jt}); \theta\right) + \sum_{t=1}^{T} \left(\ln f_i(x_{it}) + \ln f_j(x_{jt})\right) ,$$

(note that the second term does not depend on θ, nor on $type$) where $c_{type}(u_1, u_2; \theta)$ denotes the parametric expression of the density for the chosen copula ($type \in \{Cl, Gu, Fr\}$), and it records the values $\ell^*_{(i,j),type}$ and $\theta^*_{(i,j),type}$ such that

$$\ell^*_{(i,j),type} = \ell_{(i,j),type}(\theta^*_{(i,j),type}) = \max_{\theta \in \Theta} \ell_{(i,j),type}(\theta) .$$

Note that we are taking the pairs $\{(x_{it}, x_{jt}) : t = 1, \dots, T\}$ as T i.i.d. realizations drawn from the same bidimensional distribution.

3. For each possible pair (i, j) of different sub-markets, the procedure finds $\ell^*_{(i,j)}$, $\theta^*_{(i,j)}$, and $type^*_{(i,j)}$ such that

$$\ell^*_{(i,j)} = \max_{type \in \{Cl, Gu, Fr\}} \ell^*_{(i,j),type} \tag{4}$$

and $\theta^*_{(i,j)}$ and $type^*_{(i,j)}$ are the corresponding estimated parameter and the corresponding selected copula-type, respectively.

The maximization (4) corresponds to select the copula that provides the best fit according to both AIC and SIC criteria[11]. Indeed, we have the best fit at the lowest value of the quantity

$$\begin{aligned} AIC &= -2 \times (loglikelihood) + 2 \times (n.\,parameters) = -2 \times (loglikelihood) + 2 \\ SIC &= -2 \times (loglikelihood) + \ln(n.\,observations) \times (n.\,parameters) \\ &= -2 \times (loglikelihood) + \ln(T), \end{aligned}$$

respectively, and so at the highest value of the loglikelihood.

4.2 The correlation ranking

For each possible pair (i, j) of different sub-markets, the first step of the procedure has selected the copula function: indeed by the previous point we have the copula type $type^*_{(i,j)}$ and the respective parameter $\theta^*_{(i,j)}$. The goal of the second step is to produce a classification of the most dependent pairs of sub-markets. One possible choice in order to measure the strength of the dependence between two sub-markets could rely on their parameter $\theta^*_{(i,j)}$; in the case of the Archimedean family of copulas, indeed, it gives a measure of dependence between i, j. However, the theta parameter is related to the functional form of the copula and so values of the theta parameter for different copula functions are not comparable. Therefore, in order to eliminate this issue, we use the value of Kendall's tau for each pair as the criterion for the ranking. Denoting by $\tau^*_{(i,j)}$ the value of the Kendall's tau coefficient

11. For further references see Mahfoud (2012).

as a function of $\theta^*_{(i,j)}$ (see formula (7) in Appendix), the procedure continues by considering each possible pairs of different sub-markets and it splits the final ranking of the couples of sub-markets into two groups: the pairs with positive Kendall's tau dependence coefficient (i.e. $\tau^*_{(i,j)} \geq 0$) and the ones with negative dependence coefficient (i.e. $\tau^*_{(i,j)} < 0$). Finally, it returns: a decreasing ranking of the pairs of sub-markets based on $\tau^*_{(i,j)}$ for the first group, and, for the second group, an increasing ranking of the pairs based on the (negative) value of $\tau^*_{(i,j)}$. (Note that negative dependence parameter is possible only for $type \in \{Cl, Fr\}$.)

Together with the two classes of rankings (positive and negative), the procedure returns, for each pair (i, j):

- $type^*_{(i,j)}$ (based on the following code: $1 = Fr$, $2 = Gu$, $3 = Cl$),
- the value of the difference $(\theta^*_{(i,j)} - \theta_{type^*_{(i,j)},ind})$ where $\theta_{type^*_{(i,j)},ind}$ is the value for the chosen copula $type^*_{(i,j)}$ corresponding to the independence,
- the estimated value for the Kendall's tau $\tau^*_{(i,j)}$ as a function of the theta parameter for the selected copula,
- the empirical value $e\_\tau^*_{(i,j)}$ of the Kendall's tau.

4.3 Joint Probability of Distress (JPoD)

Once checked the appropriateness of the selected copula model, the analysis is carried on computing, for each couple of sub-markets, the joint probability that both of them simultaneously exhibit increments of the distress indicator above some threshold, i.e. the joint probability of distress. We calculate this probability at different marginal threshold levels. Indeed, we recall that in the Basel Vasiceck-Gordy model the choice of a certain quantile (e.g. 99.9%) for the capital charge is not related to a specific event of distress, but it is just a regulatory confidence level for the estimation of the global credit portfolio's losses. Therefore, in our context, it seems reasonable to avoid the selection of a given "distress threshold" and analyse sub-markets' joint behaviours in the right tail at different marginal levels. More precisely, if we denote by X_i and X_j the increments of the distress indicator (defined in Section 3) for the sub-market i and j respectively, then, for each pair (x, y) of real numbers, we have:

$$\begin{aligned} P(X_i > x, X_j > y) &= 1 - P(X_i \leq x \, or \, X_j \leq y) \\ &= 1 - F_i(x) - F_j(y) + P(X_i \leq x, X_j \leq y) \\ &= 1 - F_i(x) - F_j(y) + F(x, y) \\ &= 1 - F_i(x) - F_j(y) + C(F_i(x), F_j(y)), \end{aligned}$$

where F_i and F_j are the marginal cumulative distribution functions, F is the joint cumulative distribution function of the pair (i, j) and the last equality is due to equation (6) in Appendix. Thanks to this consideration, we define our Joint Probability of Distress (JPoD) as:

$$JPoD_{(i,j)} = 1 - u_i - u_j + C_{type^*_{(i,j)}}(u_i, u_j; \theta^*_{(i,j)})$$

where $u_i, u_j \in [0, 1]$ are the levels for the marginal cumulative distribution functions F_i, F_j, typically chosen equal to 90%, 95% and 99%.

5 Results

In this section we present the results obtained from the analysis of our dataset following the methodology introduced above. In the next tables, sub-markets are referred to the previous classification (see *Table 1*). Copula types are expressed as: 1 (*Frank*), 2 (*Gumbel*), and 3 (*Clayton*). In addition, *"diff_theta"* refers to the theta parameter returned once a copula type is chosen minus the value of the theta parameter for the independence case for this type of copula. Moreover, the "*e*" before the parameter refers to the empirical estimates (when no type of copula is imposed but estimates are computed on raw data). Our perimeter is composed by 25 pairs of sub-markets which show positive estimated Kendall's tau correlations and 11 sub-markets with negative values. For the sake of clarity, our choice is to consider only the first half of the rankings, i.e. the first 10 and 5 pairs for positive and negative Kendall's tau, respectively. Therefore, we focus on those pairs of sub-markets which show estimates more distant from the independent case.

Table 8: Distress indicator: *positive* Kendall's tau

Ranking	I Sub-mkt	II Sub-mkt	Copula	diff_theta	Kendall's tau	e_Kendall's tau
1	5	6	2	0.48	0.32	0.32
2	5	8	3	0.65	0.24	0.24
3	2	3	3	0.48	0.19	0.20
4	4	5	3	0.45	0.18	0.16
5	1	6	3	0.36	0.15	0.14
6	8	9	1	1.09	0.12	0.12
7	2	7	3	0.25	0.11	0.09
8	1	8	2	0.12	0.11	0.11
9	4	8	1	0.99	0.11	0.11
10	1	5	3	0.23	0.10	0.08

Table 9: Distress indicator: *negative* Kendall's tau

Ranking	I Sub-mkt	II Sub-mkt	Copula	diff_theta	Kendall's tau	e_Kendall's tau
1	5	7	1	-2.23	-0.24	-0.23
2	3	8	3	-0.33	-0.20	-0.24
3	1	3	1	-1.52	-0.16	-0.17
4	7	8	3	-0.22	-0.12	-0.06
5	6	7	1	-0.98	-0.11	-0.10

Positive Kendall's tau estimates show a very interesting behaviour if we focus on the pairs of sub-markets in the first positions of the ranking. In fact, let us rewrite the 9 sub-markets by an integer triple M_j, $j = 1...9$ as follows:

$$M_j = (f_j, t_j, c_j)$$

where

- f = frequency, 0 = 3m-3m, 1 = 6m-3m
- t = tenor range, 0 = short, 1 = medium, 2 = long

- c = clearing, 0 = cleared, 1 = un-cleared.

Hence, the sub-markets span a very simple discrete space where we could define between each couple a *Manhattan*-like distance, e.g.

$$d(M_i, M_j) \equiv |f_j - f_i| + |t_j - t_i| + |c_j - c_i| \,. \tag{5}$$

Given this simple framework we can go more in depth on the positive estimates of Kendall's tau values. In particular, we can observe that all the first four (with respect to the Kendall's tau metrics) couples have the minimum distance between their components, i.e. $\|M_j - M_i\| = 1$. This a very appealing empirical fact. Despite several issues related to the difficulty to identify sub-markets, the pioneering work on TRs data and the new distress indicator definition, these preliminary outcomes are near to the practical intuition. In addition, as shown by the *Table 9* even negative estimates can occur. This is the case of pairs of sub-markets with quite different maturities and clearing conditions. Hence, sub-markets with different features are likely to show opposite co-movements.

Finally, in both tables for the positive and negative Kendall's tau rankings (*Tables 8* and *9*) we observe that there is a very high correlation among the empirical Kendall's tau $e\_\tau^*_{(i,j)}$, which is calculated on the two vectors not processed through the copula procedure we decided to apply, and the Kendall's tau $\tau^*_{(i,j)}$ performed instead based on the type of copula chosen for the pair of sub-markets (i, j) and its estimated parameter $\theta^*_{(i,j)}$. In particular, this correlation is equal to 0.991 for positive ranking table and to 0.955 for the negative one. This suggests that the copula selection procedure provides a good fit.

Table 10: JPoD at different marginal threshold levels

JPoD ranking	tau ranking	I Sub-mkt	II Sub-mkt	copula	JPoD 90%	JPoD 95%	JPoD 99%
1	1	5	6	2	4.4963%	2.1259%	0.4059%
2	8	1	8	2	2.2563%	0.9288%	0.1540%
3	2	5	8	3	1.5457%	0.3985%	0.0164%
4	6	8	9	1	1.4875%	0.3901%	0.0163%
5	9	4	8	1	1.4381%	0.3755%	0.0156%
6	3	2	3	3	1.4142%	0.3619%	0.0148%
7	4	4	5	3	1.3843%	0.3537%	0.0144%
8	5	1	6	3	1.3089%	0.3330%	0.0135%
9	7	2	7	3	1.2176%	0.3082%	0.0124%
10	10	1	5	3	1.2032%	0.3043%	0.0123%

Moreover, we report in *Table 10* the JPoD at marginal levels for F_i and F_j both equal to 90%, 95% and 99%, respectively. In particular, since we attempt to study the joint probability of distress, we focus on those pairs of sub-markets which exhibit positive Kendall's tau values (see *Tables 8*). In addition, we select these thresholds since our aim is to provide some insights of the relationships at the tail of the distribution corresponding to deteriorated market conditions. Preliminary results suggest that for the first two pairs the JPoD assumes quite relevant values, while for the other positions estimates are almost comparable. For instance, the first pair of sub-markets in the ranking position is $(i, j) = (5, 6)$, meaning that this couple of sub-markets has the most correlated increases in terms of percentage of the distress indicators (I_i, I_j) at 90%, 95%, 99% levels for the marginal cumulative distribution functions. This couple of sub-markets shares the same swap legs (6m3m), the same cleared conditions (cleared contracts in both sub-markets), but have different maturities (Medium vs Long). In addition they represent the two most active segments in the IRS market, as reported in *Table 2*. Therefore, it seems that the two most important sub-markets in terms of number of deals and traded notional amounts are also highly co-dependent. At first glance, if we look at the following positions we can observe slightly different rankings compared to those shown in *Tables 8*. However, it is worthwhile to highlight that JPoD and Kendall's tau rankings are coherent once we focus on a certain type of copula, i.e. given the same copula we observe that the ordering for pairs of sub-markets is the same for both types of rankings. Finally, we briefly analyse the second position in *Table 10*, that is $(1, 8)$. This couple of sub-markets has different swap legs (3m3m vs 6m3m), different cleared conditions (cleared vs un-cleared), and different maturities (Short vs Medium). Still, they share a high probability of joint distress, thus supporting the need of further investigation on the features that can impact on the reciprocal influence between sub-markets apparently very distant.

6 Conclusions and future research

The financial crisis of the last decade motivated a growing literature on how to model and to predict the financial distress. Some concepts such as the *systemic risk*, the *contagion* effect and the *cascade* defaults received an increasing attention. Nevertheless, a "new normal" for the risk management field has not yet been established. If we look at the financial system as a whole, several challenging aspects have yet to be solved, such as: the huge number of risk factors and financial products, their dependence structures, the lack of complete and granular data about the financial system, the quality of available data, the measures to be used to capture and predict the financial distress.

To partially address these issues, we exploited and combined in an innovative way some new ingredients, namely the OTC derivatives data coming from the trade repositories along with the JPoD approach suggested in the recent years by the International Monetary Fund. To the best of our knowledge, this is the first attempt that exploits data from trade repositories to study co-dependence phenomena between financial sub-markets. In particular, we focused on the interest rate derivatives as a significant fraction of the OTC market and we defined a distress indicator by combining four different distress drivers, such as liquidity, average traded volumes, volatility and bid-ask proxies. Hence, we attempted to study by this framework the *distress dependencies* of some OTC sub-markets that we built according to contractual and financial features. By analysing both the descriptive results and the final joint probabilities of distress, the proposed technique seems to be quite promising. Despite its complex work flow, preliminary results are close to the practical intuition.

Some improvements should be developed in the future research. Among them, we should state the need for a sharp distress definition (or event) in order to calibrate a more general distress indicator formula which can be used to *backtesting* procedures, i.e. to assess its prediction properties. Furthermore, once other asset classes will be available in our database (equity, credit, forex, etc.) a top-down financial "classical" sub-market segmentation could be explored. Finally, some deeper knowledge about the TRs effective internal data quality is required.

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A Appendix

Copula functions provides a mathematical instrument for the modelization of the multivariate stochastic dependence structure. In particular, copulas take into account various kinds of stochastic dependence structures among actors, without any assumption on the one-dimensional marginal distributions. The concept of copula was introduced during the forties and the fifties with Hoeffding (1994) and Sklar (1959), but the evidence of a growing interest in this kind of functions in statistics started only in the nineties Nelsen (2006). Copulas are functions that join or "couple" multivariate distribution functions to their one-dimensional marginal distributions. The advantage of the copula functions and the reason why they are used in the dependence modeling is related to the Sklar's theorem Sklar (1959). It essentially states that every multivariate cumulative distribution function can be rewritten in terms of the margins, i.e. the marginal cumulative distribution functions, and a copula. More precisely, we have the following definition and results.

Definition 1. *A d-dimensional copula $C(\mathbf{u}) = C(u_1, ..., u_d)$ is a function defined on $[0,1]^d$ with values in $[0,1]$, which satisfies the following three properties:*

1. *$C(1, ...1, u_i, 1, ..., 1) = u_i$ for every $i \in \{1, ..., d\}$ and $u_i \in [0,1]$;*
2. *if $u_i = 0$ for at least one i, then $C(u_1, \ldots, u_d) = 0$;*
3. *for every $(a_1, ..., a_d), (b_1, ...b_d) \in [0,1]^d$ with $a_i \leq b_i$ for all i,*

$$\sum_{j_1=1}^{2} ... \sum_{j_d=1}^{2} (-1)^{j_1+...+j_d} C(u_{1,j_1}, ..., u_{d,j_d}) \geq 0$$

where, for each i, $u_{i,1} = a_i$ and $u_{i,2} = b_i$.

Theorem 1. *Let F be a multivariate cumulative distribution function with margins $F_1, ..., F_d$. Then there exists a copula $C : [0,1]^d \to [0,1]$ such that, for every $x_1, ..., x_d \in \overline{\mathbb{R}} = [-\infty, +\infty]$, we have*

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d)). \tag{6}$$

If the margins $F_1, \ldots, F_d$ are all continuous, then C is unique; otherwise C is uniquely determined on $F_1(\overline{\mathbb{R}}) \times \cdots \times F_d(\overline{\mathbb{R}})$.

Conversely, if C is a copula and $F_1, \ldots, F_d$ are cumulative distribution functions, then F defined by (6) is a multivariate cumulative distribution function with margins $F_1, \ldots, F_d$.

In the case when f and $f_1, \ldots, f_d$ are the marginal probability density functions associated to F and $F_1, \ldots, F_d$, respectively, the copula density c satisfies

$$f(x_1, \ldots, x_d) = c\big(F_1(x_1), \ldots, F_d(x_d)\big) \prod_{i=1}^{d} f_i(x_i).$$

There are different families of copula functions that capture different aspects of the dependence structure: positive and negative dependence, symmetry, heaviness of tail dependence and so on. In our work, we limit ourselves to the principal copula functions of the Archimedean family (namely, Gumbel, Clayton and Frank copulas), which model, through a unique parameter θ, situations with different degrees of dependence.

For more details on copula theory, we refer to the various excellent monographs existing in literature, such as Joe (1997), Nelsen (2006) and Trivedi and Zimmer (2007).

A.1 Archimedean family of copulas

Here we just recall, in the bivariate case, the principal copula functions belonging to the Archimedean family that we employ in our analysis Huynh, Kreinovich, and Sriboonchitta (2014).

- **Frank copula**:

$$C_{Fr}(u_1, u_2; \theta) = -\frac{1}{\theta} \ln \left(1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{\exp(-\theta) - 1} \right), \qquad \theta \in \Theta = (-\infty, +\infty) \setminus \{0\}.$$

  The parameter θ tunes the degree of the dependence. The limiting cases $\theta \to \theta_{Fr,ind} = 0$ correspond to independence.

- **Gumbel copula**:

$$C_{Gu}(u_1, u_2; \theta) = \exp \left\{ - \left[(-\ln u_1)^{\theta} + (-\ln u_2)^{\theta} \right]^{\frac{1}{\theta}} \right\}, \qquad \theta \in \Theta = [1, +\infty).$$

  The parameter θ tunes the degree of the dependence. In particular, the value $\theta = \theta_{Gu,ind} = 1$ corresponds to independence (indeed, we get $C^{Gu}(\mathbf{u}; 1) = \prod_{i=1}^{d} u_i$).

- **Clayton copula**:

$$C_{Cl}(u_1, u_2; \theta) = \left(u_1^{-\theta} + u_2^{-\theta} - 1 \right)^{-\frac{1}{\theta}}, \qquad \theta \in \Theta = [-1, +\infty) \setminus \{0\}.$$

  The parameter θ controls the degree of the dependence. The limiting case $\theta \to \theta_{Cl,ind} = 0$ corresponds to independence.

A.2 Kendall's tau

Consider two random variables X, Y with continuous marginals F_1, F_2 and joint cumulative distribution function F [12]. The Kendall's tau correlation coefficient is defined as:

$$\tau(X, Y) = P\{(X_1 - X_2)(Y_1 - Y_2) > 0\} - P\{(X_1 - X_2)(Y_1 - Y_2) < 0\}$$

where (X_1, Y_1) and (X_2, Y_2) are two independent pairs of random variables from the joint distribution F. It can be written in terms of the copula function as follows:

$$\tau(X, Y) = 4 \int_0^1 \int_0^1 C(u_1, u_2)\, dC(u_1, u_2) - 1.$$

In particular, for the Archimedean copulas, the Kendall's tau can be expressed as a function of the dependence parameter θ:

$$\tau(X, Y) = \begin{cases} 1 + 4\theta^{-1}[\theta^{-1} \int_0^{\theta} t/(e^t - 1)\, dt - 1] & \text{Frank} \\ 1 - \theta^{-1} & \text{Gumbel} \\ \theta/(\theta + 2) & \text{Clayton.} \end{cases} \tag{7}$$

12. For further details, see Trivedi and Zimmer (2007).

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