Labor Supply and Capital Accumulation in an Aging Economy: When Beveridge meets Bismarck[1]

Xue Wen[2], Fabio Pammolli[3], Giorgio Gnecco[4]

IMT Institute for Advanced Studies, Lucca, Italy

Abstract

In the context of an aging economy, the question addressed in this paper is: since pension systems differ in the funding methods - pay-as-you-go (PAYG) or fully funded - and payment schemes - Beveridgean or Bismarckian - under which setting can a sustainable public pension system provide both intergenerational and intragenerational redistribution, reduce labour supply distortion, and lead to a higher physical capital accumulation? Considering a series of partial reforms within a PAYG pension system to deal with aging, the results of our analysis show that commonly used policy actions distort labor supply and depress the capital market, thus, reducing the tax base and deteriorating the growth of the economy. As a consequence, the PAYG pension system does not appear to be reformable from inside, and a (partial) transition to a funded system is necessary. Moreover, we show that, within a fully funded scheme, a transition from a pure Beveridgean system to a pure Bismarckian system substantially improves the labor supply incentives, while it tends to depress physical capital accumulation. Hence, a mix between Beveridge and Bismarck substantiates a good compromise to balance the trade-off between equity and efficiency.

Keywords: **Labor Supply, Capital Accumulation, Pension, Aging Economy**

JEL Codes: H55, H21, D91

---

[1] Authors acknowledge funding from the National Research Program of Italy (PNR) project "CRISIS Lab".

[2] *E-mail address*: xue.wen@imtlucca.it. Laboratory for the Analysis of Complex Economic Systems, IMT Institute for Advanced Studies, Piazza S. Francesco 19 - 55100 Lucca, Italy.

[3] *E-mail address*: f.pammolli@imtlucca.it. Laboratory for the Analysis of Complex Economic Systems, IMT Institute for Advanced Studies, Piazza Francesco 19 - 55100 Lucca, Italy.

[4] *E-mail address*: giorgio.gnecco@imtlucca.it. Dynamical Systems, Control, and Optimization Research Unit, IMT Institute for Advanced Studies, Piazza Francesco 19 - 55100 Lucca, Italy.

1 Introduction

In face of demographic aging, the sustainability of pension systems has become the subject of a hot debate both in academia and in policy making. In general, countries implement systems consisting of three main pillars: the social security system, an occupational pension scheme, and a voluntary organized saving plan. The first pillar is mostly a PAYG system, while the last two are funded systems (see World Bank, 1994). As is well known, PAYG contributions are paid directly to the accounts of pensioners, while in a fully funded system, the contributions are invested in a fund. In addition to the distinction between PAYG and fully funded systems, pension schemes can differ in terms of the payments schemes: Beveridge or Bismarck. Beveridgean follows the flat benefit rule whereas Bismarckian the earning-related rule. The Beveridgean system is highly redistributive and achieves complete equalization of benefits, whereas no redistribution occurs in a pure Bismarckian system.[5]

In most OECD countries, public pension systems are mainly financed via PAYG, which raises the question of their sustainability in face of aging. Given its current statutory rules and the aging demographic feature, such a pension system is not financially sustainable.[6] The aging population implies the countries with a PAYG pension system need a reform that either increases contributions or reduces benefits. Commonly used policy actions such as raising taxes can possibly reduce tax base and deteriorate the growth of the economy. There is no common opinion in literature on whether the PAYG system should be replaced with a funded system.[7] Each theoretical or empirical model comes with a particular set of assumptions and motivations. Many studies have compared the present PAYG social security programs to fully or partially funded alternatives, showing their different implications for economic growth (see, e.g., Feldstein, 2005). From a theoretical perspective, consolidation or pre-funding can represent a solution to problems associated with the PAYG in aging societies.[8] de la Croix et al. (2004) discuss the optimal allocation of resources

[5] See more detailed descriptions regarding Beveridgean and Bismarckian systems in Cremer and Pestieau (2003), Conde-Ruiz and Profeta (2007), and Galasso and Profeta (2014).

[6] Based on the seminal papers from Samuelson (1958) and Aaron (1966), a PAYG pension system is financially sustainable in the long term if the product of the population growth rate and the growth rate of wages is higher than the return to capital. Other things being equal (migration, fertility, education…), in an ageing economy, the sustainability of pension systems is challenged for two reasons: on one hand, the ratio between the size of the working population contributing to the public pensions and the size of the pensioners' population increases with time, hence, to maintain the pension systems one needs to raise the pension tax rate. On the other hand, this will reinforce the labor supply distortion of the working population, thus, reduce aggregate labor productivity.

[7] Discussion on the relative merits of funded and unfunded social security has rested on the scheme satisfying the so-called "Aaron condition", that a PAYG system is more welfare improving than a FF system if the growth rate of total wage income exceeds the interest rate. Note that the Aaron condition is not applicable when some variables are endogenous, see for instance Kolmar (1997) where fertility is endogenous. See also Sinn (2000) for a detailed review on the pros and cons of a funded pension system.

[8] The ageing demographic shift has been challenging the traditional PAYG public pension system in most OECD economies. The financial crisis has deteriorated the situation further. To accomplish budget targets, short term consolidations such as raising tax or cutting public consumption have been implemented. However, there is a trade-off

across generations under falling fertility. Andersen (2008) considers the effects of rising longevity on intergenerational distribution and risk sharing, calling for increasing retirement ages to adjust to longevity. Departing from much of the literature focusing solely on intergenerational redistribution, which considers mostly the optimal pension funding method, our approach provides both an inter- and intra-generational redistribution analysis for pension systems, considering different funding methods and payment schemes: the PAYG and fully funded (FF) systems, as well as the Beveridgean (flat benefit) and Bismarckian payment schemes (contribution related).

A pension system introduces three main effects on the economy: a saving effect, a capital-labor substitution effect, and a labor distortion effect. The saving effect is extensively investigated in the literature, while the other two effects are less studied. Our analysis hence focus both on labor distortion effects and growth effects when pension systems vary in funding methods and payment schemes.

Public PAYG pension schemes have often been criticized as detrimental to growth given the standard argument that they reduce per capita income. Feldstein (1974) concludes that PAYG has a negative effect on capital accumulation since it discourages private savings. Within PAYG, payments go directly to the pensioners' accounts, and, in an aging economy, the implicit rate of return on contributions to a PAYG scheme typically falls short of the interest rate. Hence, PAYG depresses wages and income growth due to the negative effect it produces on physical capital accumulation. Moreover, we argue that adding endogenous labor supply to the standard model dampens these forces. PAYG can distort labor/leisure decision due to the fact that in an aging economy the returns to PAYG contributions are typically lower than the returns to FF contributions, since the former depend mainly on demographic factors and on economic growth of individual wages whereas the latter depend on returns and assets on capital market.

In addition, we consider another dimension of public pension systems, which is payment schemes: Beveridge or Bismarck. In an endogenous labour supply setting, Fenge (1995) and Brunner (1996) show that a shift to a pension system with a stronger contribution-benefit link (Bismarckian system) can reduce the labour-leisure distortion. However, their studies focus on Pareto efficiency and on welfare analysis instead of explaining the effects on economic growth in different systems.

In the paper we focus also on the relationship between the features of pension systems and the process of physical capital formation.[9] Our major theoretical predictions show that redistributive

between the short-term and long-term effects of such temporary policy measures. Hence, alternative policy initiatives and reforms such as pre-funding or a partially transition to a funded system to ensure fiscal sustainability are urgent. Note that the transition to pre-funding usually also requires fiscal consolidation and debt, depending on available fiscal space.

[9] It is worth mentioning that there is another trend of literature which considers the effects of pension systems on human capital accumulation growth, as social security may also affect future productivity through education and fertility choice.

pension policies depress physical capital accumulation and economic growth. The underlying mechanism is that national welfare programs such as social security systems or public health care, embodying both intergenerational and intragenerational redistribution, crowd out private savings and national investment. Sinn (2000) concludes that a partial transition from the PAYG to a funded system may be a way to overcome the current demographic crisis, because it replaces missing human capital with real capital and helps smooth tax costs across generations. We reach a similar conclusion by considering only pure pension systems (PAYG or FF), but we expect the results to be applicable to mixed pension systems as well. Moreover, since a representative agent framework does not capture intragenerational distribution, we present a model with two productivity types. We show the significance of the distortion effects is further reinforced when intragenerational redistribution is considered. However, the Bismarckian system with a strong contribution-benefit link is not necessarily favoured over the Beveridgean system with flat benefits in terms of capital growth. The underlying mechanism explaining this preference is that lower indexation of pensions on contributions leads highly productive households to increase their savings whereas the less productive agents save less because they benefit from a more generous pension. Finally, based on our assumption of equal size of highly productive and less productive labour, the total effect is positive because the positive effects on saving from the productive households over-compensate the negative effects from the less productive households. Hence, we conclude that instead of focusing on intragenerational redistribution, pension reforms that reduce intergenerational redistribution can significantly boost economic growth and total production.

Our model and simulations aim at analyzing both labor supply distortion and capital accumulation effects of pension systems that consist of intergenerational redistribution and intragenerational redistribution. Our results show that structural pension reforms such as transforming the PAYG systems to (partially) fully funded systems seem to be more efficient measures. This relates to the fact that funded system can substantially reduce labour supply distortions and lead to higher physical capital accumulation. The novelty of this paper lies in the demonstration that, while a transition from a pure Beveridgean system to a pure Bismarckian system substantially improves the labor supply incentive for the fully funded case, it may depress the physical capital accumulation. Hence, a funding system with a mix of the two payment schemes seems to be a good compromise to balance the trade-off between labor supply incentives and

For instance, Kaganovich and Zilcha (1999) and Glomm and Kaganovich (2003) study the role of education in the relationship between PAYG social security funding and economic growth. Docquier and Paddison (2003) and Lambrecht et al. (2005) compare the incentive for investment in education under PAYG and fully funded pension systems. In empirical literature, there are two opposing views regarding the relationship between redistributive expenditure and growth when one also considers human capital. Atkinson (1995) finds mixed empirical evidence on the sign of the correlation between social security and growth in OECD countries. Sala-i- Martin (1992) obtain a positive correlation between social security expenditure and cross-country growth by extending the analysis to a larger set of countries.

physical capital accumulation. Hachon (2010) reports the same finding for the PAYG system based on a model where the agent's productivity is related to his life expectancy.

The paper is organized as follows. Section 2 introduces the model setting regarding the households maximization problem (Subsection 2.1), pension system features (Subsection 2.2) and production side (Subsection 2.3), respectively. Section 3 presents the closed economy general equilibrium framework used in this paper and the main analytical findings. Section 4 gives numerical simulation results confirming our findings in Section 3. The last section provides policy implications and concludes.

2 The Model

The basic framework is an overlapping generations (OLG) model à la Diamond (1965). We consider a closed economy where firms produce a single homogenous good that can be used for both consumption and investment. The human and physical capital are used as inputs in a constant returns neoclassical technology. In a two-period OLG setting, we assume two types of individuals: low-skilled and high-skilled. Both PAYG and FF pension systems are considered, each in combination with two payment schemes: Beveridgean and Bismarckian. This section starts with Subsection 2.1 on households saving and labor supply decisions, followed by Subsection 2.2 with an analysis of government budget constraint with different pension systems in funding methods and payment schemes. Lastly in Subsection 2.3 the production function of the economy is presented.

2.1 Households Decisions

Generations are non-altruistic, implying each old generation has no bequests motive. The economy consists of two types of individuals (i): low-skilled (L) and high-skilled (H). People work in the first period of life and retire in the second period. Individuals differ in their endowment of human capital h_i, where $h_L < h_H$. The income level of an individual $y_{i,t}$ in his working period t is influenced by the wage level, his ability type and corresponding labor supply, i.e., $y_{i,t} = w_t h_i l_{i,t}$. Here w_t is the wage rate per efficient unit of labor, while $l_{i,t}$ is the labor supply provided in the working period t by an individual of type i. Both types of individuals contribute to the public pension system when young and receive pension benefits when retired. The pension contribution rate is fixed and equal to τ where $0 < \tau < 1$. During the working period, individuals make labor supply and saving decisions. The size of the young working population in period t is N_t, with the growth factor at time $t+1$ being denoted by ρ_{t+1}, i.e., $\frac{N_{t+1}}{N_t} = 1 + \rho_{t+1}$. In the following we consider the case in which $-1 < \rho_{t+1} < 0$, so the population is actually reducing from time t to time $t + 1$, which satisfies our assumption of an aging economy. For simplicity, we assume the ratio between the number of low-skilled and the number of high-skilled workers in the economy to be constant over time and equal to 1. Therefore, both groups of workers are assumed to be of equal size $\frac{N_t}{2}$ at each time.

The agents make decisions on saving and labor supply at the beginning of their first period. The preference of a type i agent living at times t and $t + 1$ is described by the life-cycle utility reported below:

$$U_{i,t} = u(c_{i,t} - l_{i,t}^2) + \beta\, u(\, c_{i,t+1}), \tag{1}$$

$$s.t.\, c_{i,t} + s_{i,t} \leq\ (1 - \tau) w_t h_i l_{i,t}, \tag{2}$$

$$s.t.\, c_{i,t+1} \leq R_{t+1} s_{i,t} + p_{i,t+1}. \tag{3}$$

Preferences are defined by the utility function u that is strictly increasing and strictly concave, which implies that, at optimality, the constraints given by Eq. (2) and (3) are satisfied with the equality. $c_{i,t}, c_{i,t+1} \geq 0$ denote the consumption levels during working and retired periods respectively, whereas $l_{i,t} \geq 0$ is the labor supply provided in the working period. Note that the quadratic disutility of labor is not crucial for the qualitative nature of the results, however, the quasi-linear specification is crucial for our results as it assumes away income effects.[10] The parameter β represents the preference for future consumption for each type of individuals. The young working generations allocate their after tax wage income $(1 - \tau) w_t h_i l_{i,t}$ between consumption $c_{i,t}$ and savings $s_{i,t}$. The old retired generation receives returns on their previous savings $s_{i,t}$ with a real interest rate r_t and pension payments $p_{i,t+1}$, which are dependent on the type. The agent's before tax income depends both on his human capital endowment and labor supply. The budget constraints of type i agents in their working and retired periods are represented by Eq. (2) and (3) respectively. For analytical tractability we consider the case of logarithmic utility afterwards. Consequently, the individual maximization problem can be written as:

$$\max_{l_{i,t}, s_{i,t}, c_{i,t}, c_{i,t+1}} U_{i,t} = \ln(c_{i,t} - l_{i,t}^2) + \beta \ln c_{i,t+1}, \tag{4}$$

subject to the constraints expressed by Eq. (2) and (3). We assume when individuals make labor supply and saving decisions in period t, they anticipate some future variables and decisions. In other words, R_{t+1} and w_t are treated as exogenous and perfectly known variables during each individual maximization, whereas the dependence of $p_{i,t+1}$ on the decision variable $l_{i,t}$ varies based on different pension systems and payment schemes. Concluding, an individual of type i born in

[10] In this setting, leisure is not taken into account as a normal good. See, e.g., Sommacal (2006) for a detailed discussion of the role of labor supply in evaluating the redistributive impact of a pension system with different utility functions.

period t is endowed with h_i units of human capital and chooses $s_{i,t}, l_{i,t}, c_{i,t}, c_{i,t+1}$ to maximize his life-cycle utility expressed by Eq. (1) under the constraints given by Eq. (2), (3). Expressing $c_{i,t}$ and $c_{i,t+1}$ as functions of $l_{i,t}$ and $s_{i,t}$ by exploiting such two constraints and assuming an interior solution for the resulting unconstrained optimization problem, one obtains the following necessary and sufficient first-order optimality conditions:

$$\frac{\partial U_{i,t}}{\partial s_{i,t}} = \frac{-1}{c_{i,t} - \frac{l_{i,t}^2}{2}} + \beta \frac{R_{t+1}}{c_{i,t+1}} = 0, \tag{5}$$

$$\frac{\partial U_{i,t}}{\partial l_{i,t}} = R_{t+1}[(1-\tau)w_t h_i - l_{i,t}] + \frac{\partial p_{i,t+1}}{\partial l_{i,t}} = 0. \tag{6}$$

Taking the other variables such as pension payments $p_{i,t+1}$, wage level w_t and interest rate r_{t+1} as given, we derive the saving level $s_{i,t}$ and individual labor supply $l_{i,t}$ for a type i agent born in period t as reported below:

$$s_{i,t} = \frac{\beta(1-\tau)w_t h_i l_{i,t} - \frac{\beta l_{i,t}^2}{2} - \frac{p_{i,t+1}}{R_{t+1}}}{1+\beta}, \tag{7}$$

$$l_{i,t} = (1-\tau)w_t h_i + \frac{\frac{\partial p_{i,t+1}}{\partial l_{i,t}}}{R_{t+1}}. \tag{8}$$

We assume saving decisions $s_{i,t}$ can be negative, due to the fact that retirees receive a pension payment out of which they can fulfill their credit obligations. Both the decisions for $s_{i,t}$ and the decisions for labor supply $l_{i,t}$ depend on the funding methods and the pension payments scheme (e.g., through the terms $p_{i,t+1}$ and $\frac{\partial p_{i,t+1}}{\partial l_{i,t}}$, which will be discussed in detail in Section 3). We first present the pension payment rules in both PAYG and FF systems in the next subsection, separately.

2.2 Pension Systems and Public Budget Constraint

Beveridgean Scheme

In a Beveridgean pension scheme, the pension payments are universal among the population. Regardless the productivity type, each individual born at time t receives the same pension benefits when retired at time $t+1$. In other words, the pension benefits one receives are not related to the contributions one made when young. We use "*Bev*" as the index for a Beveridgean pension scheme. Concluding, the pension benefits rule for the low-skilled and high-skilled workers is written as:

$$p_{L,t+1}^{Bev,PAYG} = p_{H,t+1}^{Bev,PAYG}. \tag{9}$$

Bismarckian Scheme

Instead, in a Bismarckian scheme, the pension benefits are contribution-related. Those who contribute more to the pension system during the working period receive more when retired.

Therefore, a Bismarckian scheme produces no intra-generational transfer. We use "Bis" as the index for a Bismarckian pension scheme. The Bismarckian pension payments rule for the low-skilled and high-skilled workers therefore becomes:

$$\frac{p_{L,t+1}^{Bis,PAYG}}{p_{H,t+1}^{Bis,PAYG}} = \frac{h_L l_{L,t}^{Bis,PAYG}}{h_H l_{H,t}^{Bis,PAYG}}. \tag{10}$$

PAYG

In PAYG system, government collects partially the wage income of the young working generation to pay for the pension benefits of the currently old retired generation. The key character of a PAYG system is that the contribution collected by the government goes directly to the pensioners' accounts, without being used as physical capital in the production or investment process. Therefore, we observe directly redistribution from the young to the old generations. And a PAYG pension system is dependent on demographic factors. We can express accordingly the government budget constraint of a PAYG pension system as follows:

$$p_{L,t+1}^{PAYG} + p_{H,t+1}^{PAYG} = \tau w_{t+1}^{PAYG}\left(h_L l_{L,t+1}^{PAYG} + h_H l_{H,t+1}^{PAYG}\right)(1+\rho_{t+1}). \tag{11}$$

Accordingly, the pension benefits for a type i agent under the two different payment schemes for the PAYG pension systems are expressed as follows:

Beveridgean PAYG

$$p_{i,t+1}^{Bev,PAYG} = \frac{\tau w_{t+1}^{Bev,PAYG}\left(h_L l_{L,t+1}^{Bev,PAYG} + h_H l_{H,t+1}^{Bev,PAYG}\right)(1+\rho_{t+1})}{2}. \tag{12}$$

Bismarckian PAYG

$$p_{i,t+1}^{Bis,PAYG} = \tau w_t^{Bis,PAYG}(1+\rho_{t+1}) h_i l_{i,t}^{Bis,PAYG} \Omega_{t+1}^{Bis,PAYG}. \tag{13}$$

where $\Omega_{t+1}^{Bis,PAYG}$ denotes the growth factor of the economy's per capita income at time $t+1$, which is defined as

$$\Omega_{t+1}^{Bis,PAYG} = \frac{w_{t+1}^{Bis,PAYG}\left(h_L l_{L,t+1}^{Bis,PAYG} + h_H l_{H,t+1}^{Bis,PAYG}\right)}{w_t^{Bis,PAYG}\left(h_L l_{L,t}^{Bis,PAYG} + h_H l_{H,t}^{Bis,PAYG}\right)}. \tag{14}$$

Fully Funded (FF)

While the PAYG system directly transfers the young generation's contribution to the old generation, the FF system collects the contribution and invests in the production process. Therefore, the pension benefits in the FF system are defined differently. The government budget constraint of the FF pension system follows the equation reported below:

$$p_{L,t+1}^{FF} + p_{H,t+1}^{FF} = R_{t+1}^{FF} \tau w_t^{FF}\left(h_L l_{L,t}^{FF} + h_H l_{H,t}^{FF}\right). \tag{15}$$

Accordingly, the FF system shows a link between individual contributions when young and pension benefits when retired. Hence we can express as follows the pension benefits of a type i agent under the two different payment schemes in the FF system.

Beveridgean FF

$$p_{i,t+1}^{Bev,FF} = \frac{R_{t+1}^{Bev,FF}\tau w_t^{Bev,FF}\left(h_L l_{L,t}^{Bev,FF}+h_H l_{H,t}^{Bev,FF}\right)}{2}. \tag{16}$$

Bismarckian FF

$$p_{i,t+1}^{Bis,FF} = R_{t+1}^{Bis,FF}\tau w_t^{Bis,FF} h_i l_{i,t}^{Bis,FF}. \tag{17}$$

2.3 Production

Firms produce a single homogeneous good according to a Cobb-Douglas technology exhibiting constant returns to scale. The outputs and factor markets are competitive, in which the equilibrium factor prices correspond to marginal products of inputs. The production function F of the representative firm is:

$$Y_t = F(K_t, L_t), \tag{18}$$

where Y_t is the output at time t, which can be either consumed or saved as new physical capital. We assume capital is totally depreciated in each term. L_t is the aggregate labor input, K_t is the physical capital. R_t stands for the market rental rate on capital in period t and w_t is the corresponding wage rate. The representative firm maximizes in each period its profit function, solving the following optimization problem, taking $F(K_t, L_t) = AK_t^{\alpha}L_t^{1-\alpha}$:

$$\max_{K_t,L_t} F(K_t, L_t) - w_t L_t - R_t K_t = AK_t^{\alpha}L_t^{1-\alpha} - w_t L_t - R_t K_t. \tag{19}$$

It follows by the first-order optimality condition that profit maximization equates the real interest rate and the real wage rate to the marginal product of capital and labor, respectively. Combining the representative firm's profit maximizing condition and the equilibrium condition in the labor market yields:

$$w_t = A(1-\alpha)k_t^{\alpha}, \tag{20}$$

$$R_t = (1+r_t) = A\alpha k_t^{\alpha-1}. \tag{21}$$

The labor market clearing condition yields:

$$L_t = \frac{N_t}{2}\left(h_L l_{L,t} + h_H l_{H,t}\right). \tag{22}$$

We denote by $k_t = \frac{K_t}{L_t}$ the physical to human capital ratio (capital in efficiency units), which we can express as:

$$k_t = \frac{K_t}{L_t} = \frac{K_t}{\frac{N_t}{2}(h_L l_{L,t} + h_H l_{H,t})}. \tag{23}$$

3 General Equilibrium

This section provides the full solution of the individual maximization problem described by Eq. (2)-(4) taking into consideration the factor markets and government budget constraints. In equilibrium, the aggregate amount of pension payments must equal the aggregate contribution.

Definition 1. *Given the state of agents distribution in the economy and the level of low-skilled and high-skilled human capital, a dynamic equilibrium is a sequence of individual's decisions, a sequence of factor prices, and a sequence of pension payments so that:*[11]

(i) *Individuals choose* $s_{i,t}, l_{i,t}, c_{i,t}, c_{i,t+1}$ *to solve the maximization problem described by Eq. (2)-(4), taking the factor prices as given;*

(ii) *Factor markets clearing condition holds: the factor prices are equal to their marginal products, see Eq. (20) and (21);*

(iii) *The government budget constraint is satisfied, i.e., Eq. (11) is satisfied in a PAYG system, while Eq. (15) is satisfied in a FF system. The above conditions also account for a sustainable equilibrium.*

More specifically, a dynamic equilibrium is characterized by the following features in each period t.

— Factor Market Equilibrium

Equality between demand and supply of labor and capital

In the labor market equilibrium, the aggregate labor supply in period t is the sum of the labor supply from both the high-skilled and low-skilled workers, see Eq. (22).

The capital market is fully integrated in the economy. In a PAYG pension system, the supply of capital in period $t+1$ is determined by the saving decision of the young made in period t. In a FF pension scheme, the supply of capital in period $t+1$ is made up of both private and public savings

[11] For simplicity, we take the pension tax rate as an exogenous variable.

of period t.[12] Hence **in a PAYG pension system**, the physical capital K_{t+1}^{PAYG} in period $t+1$ is the sum of the aggregate previous period private savings

$$S_{i,t} = \frac{N_t}{2}\left(s_{L,t} + s_{H,t}\right), \tag{24}$$

where $s_{i,t}$ is provided by Eq. (7). Therefore, the PAYG capital market equilibrium condition is:

$$K_{t+1}^{PAYG} = \sum_{i=L,H} \frac{N_t s_{i,t}^{PAYG}}{2}. \tag{25}$$

On the other hand, **in the FF pension system**, the aggregate physical capital consists both of private savings and public savings. Accordingly, the aggregate capital in a FF pension system is:

$$K_{t+1}^{FF} = \sum_{i=L,H} \frac{N_t s_{i,t}^{FF} + N_t \tau w_t^{FF} h_i l_{i,t}^{FF}}{2}. \tag{26}$$

For the PAYG pension system, one can determine the capital stock K_{t+1}^{PAYG} from Eq. (25), knowing the values assumed by the variables N_t and $s_{i,t}^{PAYG}$. Similarly, for the FF pension system, one can determine the capital stock K_{t+1}^{FF} from Eq. (26), knowing the values assumed by the variables $N_t, s_{i,t}^{FF}, w_t^{FF}, h_i$. Individuals choose $s_{i,t}, l_{i,t}, c_{i,t}, c_{i,t+1}$ to solve the maximization problem (1)-(3), taking the factor prices as given.

Factor Prices

The factor market equilibrium requires Eq. (20) and (21) to be satisfied.

— Individual Utility Maximization

A general expression for the saving decision $s_{i,t}$ is provided by Eq. (7). Likewise the labor supply decision, it depends on the particular funding method and payment scheme. In the next section, we focus on the labor supply $l_{i,t}$ for the type i agent in the following four pension systems: Beveridgean PAYG system, Bismarckian PAYG system, Beveridgean FF system, and Bismarckian FF system.

3.1 The Labor Supply

In the following, we express the optimal individual labor supply with respect to the four different pension systems. In the Bismarckian system, pensions are assessed on the basis of past earnings. Greater work effort by the young raises not only their current income but also leads to higher pension benefits when retired. Instead, the Beveridgean system implies a flat benefit scheme

[12] See Eq. (11) and (15).

where the benefits are universal among individuals with different productivities. On the other hand, the PAYG pension system indicates that the pension payments for the old generation are paid through labor taxation on the current young generation. In a fully funded pension system, the pension payments are based on the market return of the private pension fund accounts, which means that one's pension benefits in his second period of life are related on his first period own contribution. Therefore, the labor distortion effects vary with the nature of the four different pension systems. Here, we report the labor supply $l_{i,t}$ for each pension system. Details on the derivations of such results are given in the appendix.

Beveridgean PAYG

$$l_{i,t}^{Bev,PAYG} = (1-\tau) w_t^{Bev,PAYG} h_i. \tag{27}$$

Bismarckian PAYG

$$l_{i,t}^{Bis,PAYG} = (1-\tau) w_t^{Bis,PAYG} h_i + \frac{(1-\alpha)\tau h_i k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1}) \frac{h_L l_{L,t+1}^{Bis,PAYG} + h_H l_{H,t+1}^{Bis,PAYG}}{h_L l_{L,t}^{Bis,PAYG} + h_H l_{H,t}^{Bis,PAYG}}. \tag{28}$$

Beveridgean FF

$$l_{i,t}^{Bev,FF} = (1-\tau) w_t^{Bev,FF} h_i. \tag{29}$$

Bismarckian FF

$$l_{i,t}^{Bis,FF} = w_t^{Bis,FF} h_i. \tag{30}$$

One can notice that the ratio between the labor supply choices of the high-skilled and low-skilled individuals is equal to $\frac{h_H}{h_L}$. As it is shown in the appendix, for each of the four pension systems considered in the paper, when we consider constant population growth rate where ρ_{t+1} is equal to a constant ρ, one obtains a unique nontrivial steady state solution (i.e., one characterized by non-zero values of the labor supply decisions, and of the capital in efficiency units). Hence, we provide the following expressions for the steady state values of the labor supply decisions of both types of individuals.[13]

Beveridgean PAYG

$$l_i^{Bev,PAYG} = (1-\tau) A (1-\alpha) \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_i. \tag{31}$$

Bismarckian PAYG

$$l_i^{Bis,PAYG} = (1-\tau) A (1-\alpha) \frac{\beta\tau(1-\alpha) + 2\alpha(1+\beta) + \tau(1-\alpha)(2+\beta)}{2\alpha(1+\beta) + \tau(1-\alpha)(2+\beta)}$$
$$\times \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_i. \tag{32}$$

Beveridgean FF

[13] See the derivations in the appendix.

$$l_i^{Bev,FF} = (1-\tau)A(1-\alpha)\left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_i. \tag{33}$$

Bismarckian FF

$$l_i^{Bis,FF} = A(1-\alpha)\left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_i. \tag{34}$$

Proposition 1: *Assume that $U(x) = \ln x$. In a closed country OLG model, when we consider an aging economy,*

(i) in the FF case, assuming $\alpha \leq \frac{1}{2}$, the degree of labor supply distortion at the steady state is always higher under the Bismarckian payment scheme than under the Beveridgean payment scheme;

(ii) for each type of individuals, the partial derivative of the steady state labor supply with respect to ρ is negative in all pension systems, whereas its partial derivative with respect to τ is negative for the Beveridgean PAYG system, and is equal to 0 for the Bismarckian FF system.

Proof. *(i)* A comparison of Eq. (33) and (34) shows that

$$l_i^{Bev,FF} \leq l_i^{Bis,FF}, \tag{35}$$

provided $\alpha \leq \frac{1}{2}$. Indeed, in that case, one has

$$(1-\tau)(1+\tau)^{\alpha/(1-\alpha)} = (1-\tau^2)(1+\tau)^{(2\alpha-1)/(1-\alpha)} \leq 1. \tag{36}$$

(ii) Closed-form expressions of all the partial derivatives (from which one concludes about their signs) are reported in the appendix. We summarize the results in Table 1. We use the symbol "?" to denote that the sign of the partial derivative depends on the choice of the parameters.

	Beveridgean PAYG	*Bismarckian PAYG*	*Beveridgean FF*	*Bismarckian FF*
$\frac{\partial l_i}{\partial \rho}$	–	–	–	–
$\frac{\partial l_i}{\partial \tau}$	–	*?*	*?*	0

Table 1: the signs of the partial derivatives of the steady state labor supply with respect to ρ and τ.

One can observe that Proposition 1 (ii) does not report the sign of the partial derivative of the steady state labor supply with respect to τ for the Beveridgean FF system and the Bismarckian PAYG system, as it can be either positive or negative, depending on the choices of the parameters. One can also notice that, even in the FF case, the product $w_t L_t$ is not necessarily higher in the

Bismarckian pension system than in the Beveridgean pension system, given that the wage rates in the two systems are different at equilibrium. Hence, we show and compare the steady state labor supply in the numerical simulations section.

An analytical comparison of all cases described by Eq. (31)-(34) is not straightforward. However, a numerical investigation of Eq. (31)-(34) for realistic values of their parameters is done in Section 4, and shows that the statement of Proposition 1 (i) holds also for the PAYG system, for such choices of the parameters. One can also notice that the result of such a comparison depends essentially on the parameters α, β, and τ, since Eq. (31)-(34) depend in the same ways on A and ρ. Finally, it is interesting to observe that, for the Bismarckian FF pension system, there is no dependence from τ of the steady state values of the low-skilled and high-skilled individual labor supply choices. Such a system has no labor supply distortion.

3.2 Capital Accumulation Levels in Efficiency Units

Following the equilibrium capital market conditions given before for both the PAYG and FF systems [see Eq. (22) and (23)], using the saving $\boldsymbol{s_{i,t}}$ from Eq. (7) and the pension schemes given by Eq. (11), (12), (14), (15), we can express as follows the capital in efficiency units under the four different pension schemes. Details on the derivations of such results are given in the appendix.

Beveridgean PAYG

$$k_{t+1}^{Bev,PAYG} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}} \left(k_t^{Bev,PAYG}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{39}$$

Bismarckian PAYG

$$k_{t+1}^{Bis,PAYG} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}} \left(k_t^{Bis,PAYG}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{40}$$

Beveridgean FF

$$k_{t+1}^{Bev,FF} = \left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}} \left(k_t^{Bev,FF}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{41}$$

Bismarckian FF

$$k_{t+1}^{Bis,FF} = \left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}} \left(k_t^{Bis,FF}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{42}$$

3.3 Steady State Capital Stock in Efficiency Units

In this section we address the implications of the four different pension systems on the steady state capital stock. The steady state capital stock in efficiency units for the four different pension systems is obtained from Eq. (43)-(46).[14]

Beveridgean PAYG

$$k^{Bev,PAYG} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{43}$$

Bismarckian PAYG

$$k^{Bis,PAYG} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{44}$$

Beveridgean FF

$$k^{Bev,FF} = \left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{45}$$

Bismarckian FF

$$k^{Bis,FF} = \left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{46}$$

Proposition 2*: Assume that* $U(x) = \ln x$. *In a closed economy OLG model, when we consider an aging economy,*

(i) the steady state value of the capital stock in efficiency units is always higher in the FF pension systems than in the PAYG systems. Moreover, under the same funding method, the steady state capital stock under the Beveridgean payment scheme is higher than under the Bismarckian payment scheme;

(ii) the partial derivative of the steady state value of the capital stock in efficiency units with respect to ρ *is negative in all pension systems, whereas its partial derivative with respect to* τ *is negative for the two PAYG systems, equal to 0 for the Bismarckian FF system, and positive for the Beveridgean FF system.*

Proof. *(i)* A direct comparison of Eq. (43)-(46) shows that

$$k^{Bis,PAYG} \leq k^{Bev,PAYG} \leq k^{Bis,FF} \leq k^{Bev,FF}, \tag{47}$$

for all the possible choices of the parameters.

[14] Details are provided in the appendix.

(ii) Closed-form expressions of all the other partial derivatives (from which one concludes about their signs) are reported in the appendix. We summarize the results in Table 2.

	Beveridgean PAYG	*Bismarckian PAYG*	*Beveridgean FF*	*Bismarckian FF*
$\frac{\partial k}{\partial \rho}$	$-$	$-$	$-$	$-$
$\frac{\partial k}{\partial \tau}$	$-$	$-$	+	0

Table 2: the signs of the partial derivatives of the steady state value of the capital stock in efficiency units with respect to ρ and τ.

In the appendix, it is also shown that all the nontrivial steady state solutions (43)-(46) are globally asymptotically stable. This motivates the importance of such solutions for our analysis.

4 Numerical Simulations

In this section, we show numerically how the steady state labor supply and the steady state capital stock in efficient units changes with respect to the population growth rate and the labour tax rate for the four different pension systems. In short, the numerical simulations support the theoretical findings in proposition 1 and 2. Moreover, since the sign of the partial derivative of labor supply with respect to τ is ambiguous for the Beveridgean FF and Bismarckian PAYG cases, we test, numerically, the signs for these two cases. The main results are reported in proposition 3.

Figure 1 shows, for all the four pension systems, the nontrivial steady state values for the low-skilled individual labor supply choice and the high-skilled individual labor supply choice, expressed as functions of the parameters ρ and τ, for fixed values of the other parameters α, β, A, h_L, and h_H (the choices $\beta = 0.96$, $\alpha = 0.29$, $A = 8$, $h_L = 0.5$, $h_H = 1$ have been made to generate the figure)[15]. The parameters ρ and τ range, respectively, on the intervals $[-0.5, 0)$ and $[0.2, 0.5]$. In particular, the figure shows that - the other things being equal - the nontrivial steady state values obtained in the case of the fully funded methods for the low-skilled individual labor supply choice and the high-skilled individual labor supply choice, are always larger than the corresponding ones obtained in the case of the PAYG methods. Figure 2 does a similar comparison for the capital in efficiency units.

[15] The values of these variables follow Bouzahzaha, de la Croix and Docquier (2002).

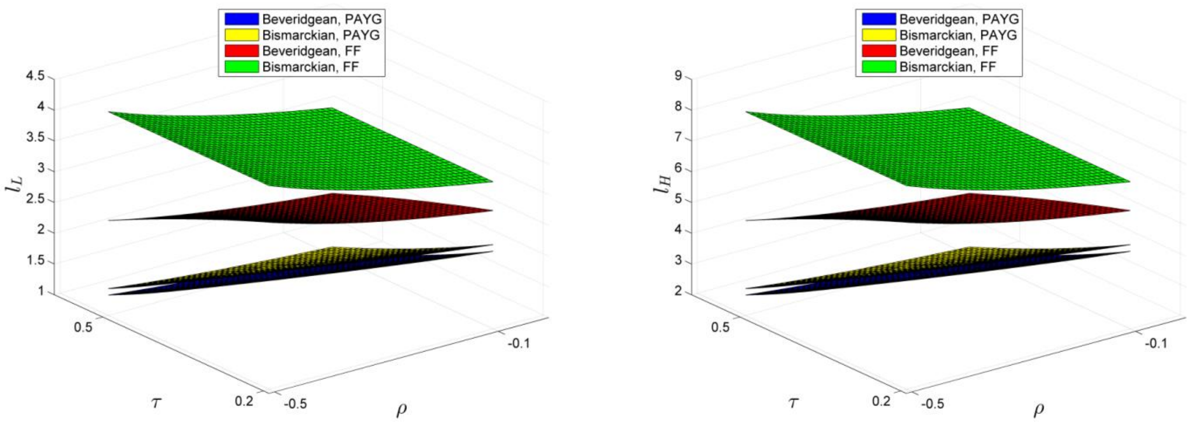

Figure 1. Nontrivial steady state values for the low-skilled (left) and high-skilled (right) individual labor supply choice for all the four pension systems studied in the paper, as functions of the parameters ρ and τ, for fixed values of the other parameters α, β, A, h_L, and h_H.

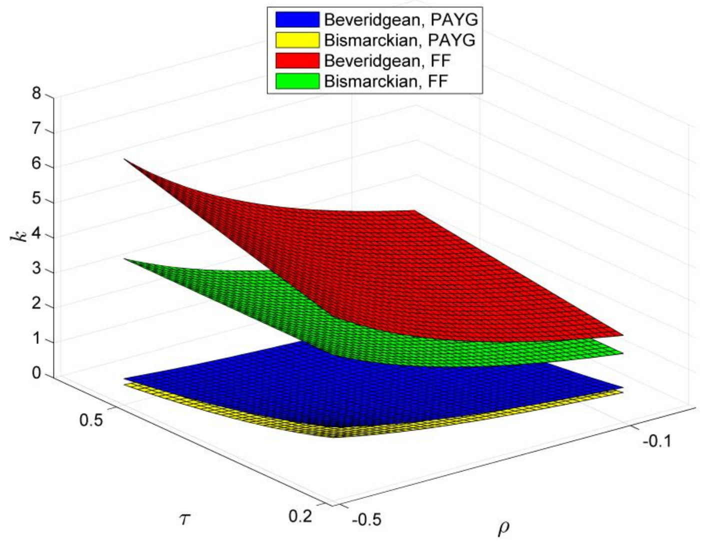

Figure 2. Nontrivial steady state values for the capital in efficiency units for all the four pension systems studied in the paper, as functions of the parameters ρ and τ, for fixed values of the other parameters α, β, A, h_L, and h_H.

It is also interesting to compare the steady state solutions obtained for the four pension systems in terms of the capital per person, defined as

$$\frac{K_t}{N_t} = \frac{h_L l_{L,t} + h_H l_{H,t}}{2} k_t, \tag{50}$$

which we evaluate, again, at the steady state values for $l_{L,t}$, $l_{H,t}$, and k_t. Figure 3 shows the results of this comparison, for the same values of the parameters as in Figure 1. We can see that, for the selected parameters, the steady state values of the capital per person for the FF pension systems are larger than the ones obtained for the PAYG pension systems.

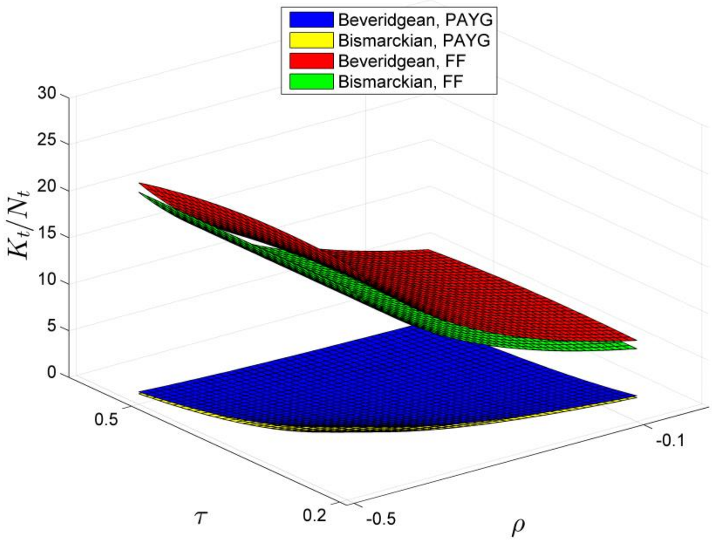

Figure 3. Nontrivial steady state values for the capital per person for all the four pension systems studied in the paper, as functions of the parameters ρ and τ, for fixed values of the other parameters α, β, A, h_L, and h_H.

Finally, Figures 4, 5, and 6 show, respectively, the portions of the plots in Figures 1, 2, and 3 that are obtained by setting the parameter ρ to -0.3.

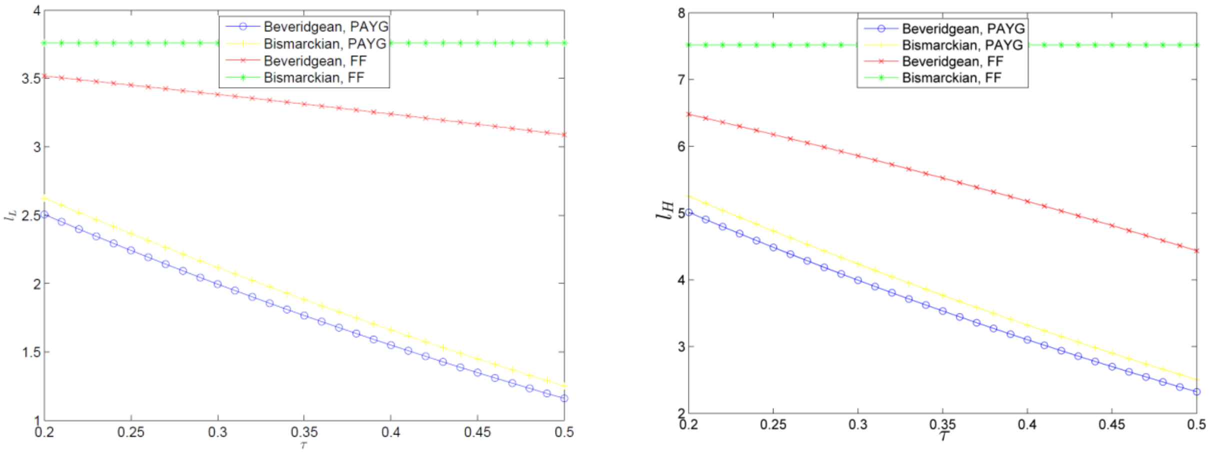

Figure 4. Nontrivial steady state values for the low-skilled (left) and high-skilled (right) individual labor supply choice for all the four pension systems studied in the paper, as functions of the parameter τ, for fixed values of the other parameters α, β, ρ, A, h_L, and h_H.

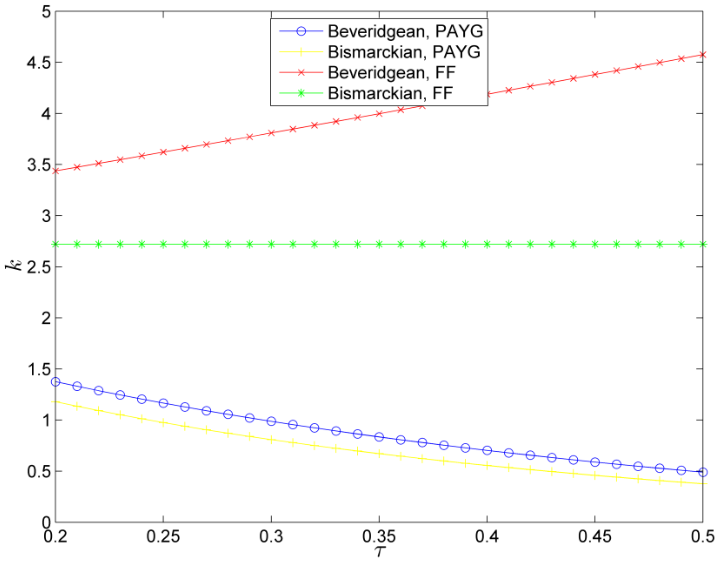

Figure 5. Nontrivial steady state values for the capital in efficiency units for all the four pension systems studied in the paper, as functions of the parameter τ, for fixed values of the other parameters α, β, ρ, A, h_L, and h_H.

We can summarize the results of the numerical comparison done in this section in the following proposition, whose proof is evident from the plots in Figures 1-3.

Proposition 3: *Assume that $U(x) = \ln x$. In a closed country OLG model, when we consider an aging economy, under the realistic choices of the parameters considered in this section,*[16]

(i) the degree of labor supply distortion at the steady state is always higher for the PAYG case than for the FF case; for both the PAYG and FF cases, the degree of labor supply distortion at the steady state is always higher for the Bismarckian pension system than for the Beveridgean pension system;

(ii) for each type of individuals, the partial derivative of the steady state labor supply with respect to ρ is negative in all pension systems, whereas its partial derivative with respect to τ is negative for both the PAYG systems and the Beveridgean FF system, and is equal to 0 for the Bismarckian FF system;

(iii) the steady state values of the capital stock in efficiency units and the capital per person are always higher in the FF pension systems than in the PAYG systems. Moreover, under the same funding method, such steady state values are higher under the Beveridgean payment scheme than under the Bismarckian payment scheme;

[16] I.e., $\beta = 0.96$, $\alpha = 0.29$, $A = 8$, $h_L = 0.5$, $h_H = 1$, whereas the parameters ρ and τ range, respectively, on the intervals $[-0.5, 0)$ and $[0.2, 0.5]$.

(iv) *the partial derivative of the steady state value of the capital stock in efficiency units with respect to ρ is negative in all pension systems, whereas its partial derivative with respect to τ is negative for the two PAYG systems, equal to 0 for the Bismarckian FF system, and positive for the Beveridgean FF system*.

	Beveridgean PAYG	*Bismarckian PAYG*	*Beveridgean FF*	*Bismarckian FF*
$\frac{\partial l_i}{\partial \rho}$	−	−	−	−
$\frac{\partial l_i}{\partial \tau}$	−	−	−	0
$\frac{\partial k}{\partial \rho}$	−	−	−	−
$\frac{\partial k}{\partial \tau}$	−	−	+	0

Table 3: For $\beta = 0.96$, $\alpha = 0.29$, $A = 8$, $h_L = 0.5$, $h_H = 1$, $\rho \in [-0.5, 0)$, and $\tau \in [0.2, 0.5]$: the signs of the partial derivatives of the steady state values of the labor supply and the capital stock in efficiency units with respect to ρ and τ.

To conclude, the simulation results summarized in Proposition 3 are consistent with the findings we provided in Section 3. For realistic values of the parameters considered in our numerical comparison, we can firstly conclude that the labor supply distortion increases when the pension tax rate is higher for all the cases except for the Bismarckian FF case, where the labor supply distortion is always zero. Moreover, a transition from a PAYG to a (partially) fully funded system substantially decreases the labor supply distortion. At the same time, a reform towards a Bismarckian system with a stronger contribution-benefit link reduces labor supply but not necessarily boosts capital stock.

5 Policy Implications and Conclusion

Reforms of the current public pension systems are continuously in hot debate in the main OECD countries, where the population is aging. Empirical evidence from 20 OECD countries shows that the overall size of the pension program increases due to the size of the ratio of the aged (over 60) to the middle aged (40-60). In particular, in most European countries, such as Italy and Germany, where the fertility rate is lower than the replacement level, the present PAYG financing of public pension system burden on active cohorts and cannot provide an adequate level of pension benefits for future cohort of pensioners. Therefore it is believed, both in academia and in policy, that substantial reforms are urgent due to the fact that the system is on the verge of collapse.[17]

This paper considers a general equilibrium model in a closed economy with an aging population. We are interested in investigating labor supply distortion effects and economic growth effects with respect to different funding methods and payment schemes of the pension system.

It is well known that, in an aging economy, a fully funded system boosts higher physical capital accumulation and therefore leads to higher economic growth.[18] The results from this paper show the same conclusion as a fully funded system leads to higher capital accumulation. When we consider the closed economy equilibrium, higher capital accumulation results in higher wages, but lower rates of return on capital. Moreover, we are not only interested in the effects of different pension systems on savings, but also on labor supply. We investigate an endogenous decision for labor supply based on four pension systems that differ in two dimensions: the funding system and the payment scheme. Under both PAYG and fully funded systems, labor supply is less distorted when the pension payment scheme is Bismarckian. In fact, labor supply is not distorted at all in the Bismarckian fully funded pension system. The model also shows in Figure 4 that labor supply of the high-skilled workers is more distorted than that of the low-skilled workers due to the fact that the high-skilled workers are endowed with higher human capital.

One aim of enacting social security reforms is to raise national savings. An argument often made in favor of funded systems is that they would accumulate a higher increase in national savings than the one that would occur with the PAYG financing channel. The effects of a reform plan on capital accumulation are usually evaluated based on the individuals' saving decisions and national saving account, which involves the direct effects on saving from change to benefits and revenue, and the indirect effects that are more complicated. For instance, the individuals may raise or reduce private savings in response to a reform. Given that we consider labor-leisure distortion effects, a reform may not only have impact on the individuals' saving decisions per se, but also influence the total wage income that indirectly offsets private savings. We show that in the demographic trend of aging, commonly used policy actions, such as raising tax rates distort labor supply and the capital market, reduce the tax base, and deteriorate the growth of the economy. Instead, structural pension reforms such as transforming PAYG systems to (partially) fully funded systems can substantially reduce labor supply distortion and boost capital accumulation. On the other hand, our model shows that even though a reform from a pure Beveridgean system to a pure Bismarckian system substantially improves the labor supply incentives, it tends to depress physical capital accumulation. Hence, we show why a funding system with a mix of the two payment schemes is a good

---

[17] See for instance, Kotlikoff (2013) that provides policy analysis for US.

[18] See in a similar framework, Docquier and Paddison (2003) shows that growth can only be stimulated under a fully funded scheme based on a Bismarckian regime depending on one's partial earnings history.

compromise to balance the trade-off between labor supply incentive and physical capital accumulation.

Note that our results are based on theoretical predictions and are purely descriptive. However, our results can still shed some light on PAYG pension reform discussions Since the seminar contribution from World Bank (1994) that proposed a multi-pillar pension scheme where publicly managed, unfunded defined-benefit (DB) schemes are shifted to privately managed, fully funded defined-contribution (DC) schemes, many countries caught on this policy vision: between 1988 and 2008, 29 countries introduced systemic reforms involving the establishment of a main funded pension pillar but with variations in design, implementation, and outcome (see for instance recent discussions of Feldstein, 2005). Taking Germany as an example, the 2001 major reform bill shifted the pure PAYG system to a multi-pillar pension system with a small but growing pre-funded pillar. In 2004, another reform transformed the PAYG pillar into a notional defined contribution (NDC) by introducing a sustainability factor into the benefit indexation formula and recommended an increase in the normal retirement age (See Börsch-Supan and Wilke, 2004). The main contribution of our paper is to propose a private fully funded system with a mix of Beveridge and Bismarck, solving the equity-efficiency tradeoff problem. Our main concern is that, a fully transition to a funded system is hard to implement due to economic and politic challenges. Not to mention that the private market fails to provide the basic Beveridgean pension because private market cannot redistribution. Therefore, a possible solution is to follow the german practice, distribute the first basic flat pension via public programs and manage the second private pension funds via financial intermediaries.

Finally, it is worth mentioning that the effects of the pension system on human capital accumulation were not analyzed in our model and deserve further investigation. A possible extension would be to introduce education as a determinant of productivity. In theory, the negative effects of redistributive pension policies on growth should be amplified vis-à-vis what we have already shown in our analysis. So our results will hold, a fortiori, when we consider human capital accumulation, since low saving rates provide poor pools of capital for investment in education for young generation if private education were not taken into consideration.

References

Aaron, H. J. 1966. The social insurance paradox, *Canadian Journal of Economics*, 32 (3), 371–374.

Andersen, T., 2008. Increasing longevity and social security reforms. A legislative procedure approach, *Journal of Public Economics*, 92 (3-4), 633–646.

Andersen, T., Bhattacharya, J., 2013. Unfunded pensions and endogenous labour supply. *Macroeconomic Dynamics*, 17 (5), 971–997.

Atkinson, A. B., 1995. The welfare state and economic performance. *National Tax Journal*, 48, 171–198.

Beltrametti, L., Bonatti, L., 2004. Does international coordination of pension policies boost capital accumulation? *Journal of Public Economics*, 88 (1-2), 113–130.

Börsch-Supan, A. and Wilke, C. B.: 2004, The German Public Pension System: How it was, How it will be, NBER Working Paper 10525.

Bouzahzah, M., de la Croix, D., and Docquier, F., 2002. Policy reforms and growth in computable OLG economies, *Journal of Economic Dynamics and Control*, 26, 2093–2113.

Breyer, F., Straub, M., 1993. Welfare effects of unfunded pension systems when labor supply is endogenous. *Journal of Public Economics*, 50 (1), 77–91.

Brunner, Johann K., 1996. Transition from a Pay-As-You-Go to a fully funded pension system: the case of differing individuals and intragenerational fairness, *Journal of Public Economics*, 60 (1), 131–46.

Cigno, A., 2008. Is there a social security tax wedge?, *Labour Economics*, 15 (1), 68–77.

Conde-Ruiz, I., Profeta, P., 2007. The redistributive design of social security systems, *Economic Journal*, 117 (520), 686–712.

Cremer, H., Pestieau, P., 2003. Social Insurance Competition between Bismarck and Beveridge. *Journal of Urban Economics,* 54, 181-196.

de la Croix, D., Michel, P., 2002. *A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations*, Cambridge, Cambridge University Press.

Disney, R., 2004. Are contributions to public pension programmes a tax on employment?, *Economic Policy,* 19 (39), 267–311.

Diamond, P. 1965. National debt in a neoclassical growth model, *American Economic Review,* 55 (5), 1126–1150.

Docquier, F., Paddison, O., 2003. Social security benefit rules, growth and inequality. *Journal of Macroeconomics,* 25 (1), 47–71.

Feldstein, M. S., 1974. Social security, induced retirement, and aggregate capital accumulation, *Journal of Political Economy*, 82 (5), 905–926.

Feldstein, M. S., 2005. Structural reform of social security, *Journal of Economic Perspectives,* 19 (2) , 33–55.

Fenge, R., 1995. Pareto-efficiency of the pay-as-you-go pension system with intragenerational fairness, *Finanzarchiv*, 52 (3), 357–364.

Galasso, V. and Profeta, P. 2014. 'Population ageing and the size of the welfare state', in *International handbook on ageing and public policy*, Cheltenham, Edward Elgar, 74- 83.

Glomm, G., Kaganovich, M., 2003. Distributional effects of public education in an economy with public pensions, *International Economic Review,* 44 (3), 917–937.

Hachon, C., 2010. Do Beveridgian pension systems increase growth?, *Journal of Population Economics*, 23 (2), 825–831

Kaganovich, M., Zilcha, I., 1999. Education, social security, and growth, *Journal of Public Economics,* 71 (2), 289–309.

Kolmar, M. 1997, Intergenerational Redistribution in a Small Open Economy with Endogenous Fertility, *Journal of Population Economics*, 10, 335-356.

Kotlikoff, Laurence J. 2013, The US fiscal cliffs – when economists recklessly endanger the economy, *Cesife Forum*, 4 (2), 3-8.

Lambrecht, S., Michel, P., Vidal, J.-P., 2005. Public pensions and growth, *European Economic Review,* 49 (5), 1261–1281.

Perotti, R., 2001. Is a uniform social policy better? Fiscal federalism and factor mobility, *American Economic Review*, 91 (3), 596–610.

Sala-i-Martin, X., 1992. *Public Welfare and Growth*, Yale University, Economic Growth Center, Discussion Paper 666, 32 pages.

Sinn, H.-W., 2000. Pension reform and demographic crisis. Why a funded system is needed and why it is not needed, *International Tax and Public Finance,* 7, 389–410.

Sommacal, A., 2006. Pension systems and intragenenerational redistribution when labor supply is endogenous, *Oxford Economic Papers*, 58 (3), 379–406.

World Bank, 1994. *World Development Report 1994: Infrastructure for Development,* Washington, DC: The World Bank.

Appendix

In this appendix, we derive recursive formulas for the capital in efficiency units, for all the pension systems investigated in the paper. We also provide the expressions of their nontrivial steady state values, which are obtained from such recursive formulas, together with the expressions of the nontrivial steady state values for the labor supply decisions and for the capital per person.

Beveridgean PAYG System

Combining Eq. (8) and (12), one obtains

$$l_{i,t}^{Bev,PAYG} = (1-\tau) w_t^{Bev,PAYG} h_i. \tag{A1}$$

This, combined with Eq. (7) and (12) again, provides

$$\begin{aligned} s_{i,t}^{Bev,PAYG} &= \frac{\beta(1-\tau)^2\left(w_t^{Bev,PAYG}\right)^2 h_i^2}{1+\beta} - \frac{\beta(1-\tau)^2\left(w_t^{Bev,PAYG}\right)^2 h_i^2}{2(1+\beta)} \\ &- \frac{\tau w_{t+1}^{Bev,PAYG}\left(h_L l_{L,t+1}^{Bev,PAYG} + h_H l_{H,t+1}^{Bev,PAYG}\right)(1+\rho_{t+1})}{2R_{t+1}^{Bev,PAYG}(1+\beta)} \\ &= \frac{\beta(1-\tau)^2\left(w_t^{Bev,PAYG}\right)^2 h_i^2}{2(1+\beta)} \\ &- \frac{\tau A(1-\alpha)\left(k_{t+1}^{Bev,PAYG}\right)^{\alpha} N_{t+1}\left(h_L l_{L,t+1}^{Bev,PAYG} + h_H l_{H,t+1}^{Bev,PAYG}\right)(1+\rho_{t+1})}{2(1+\beta)N_{t+1}A\alpha\left(k_{t+1}^{Bev,PAYG}\right)^{\alpha-1}} \\ &= \frac{\beta(1-\tau)^2\left(w_t^{Bev,PAYG}\right)^2 h_i^2}{2(1+\beta)} - \frac{\tau(1-\alpha)L_{t+1}^{Bev,PAYG}(1+\rho_{t+1})}{(1+\beta)N_{t+1}\alpha\frac{L_{t+1}^{Bev,PAYG}}{K_{t+1}^{Bev,PAYG}}} \\ &= \frac{\beta(1-\tau)^2\left(w_t^{Bev,PAYG}\right)^2 h_i^2}{2(1+\beta)} - \frac{\tau(1-\alpha)K_{t+1}^{Bev,PAYG}}{\alpha(1+\beta)N_t}. \end{aligned} \tag{A2}$$

Then, using Eq. (24), (22), and (23), one gets

$$K_{t+1}^{Bev,PAYG} = \frac{N_t}{2}\sum_{i=L,H} s_{i,t}^{Bev,PAYG} = \frac{N_t\beta(1-\tau)^2\left(w_t^{Bev,PAYG}\right)^2(h_L^2+h_H^2)}{4(1+\beta)} - \frac{\tau(1-\alpha)K_{t+1}^{Bev,PAYG}}{\alpha(1+\beta)}, \tag{A3}$$

$$K_{t+1}^{Bev,PAYG} = \frac{\alpha N_t \beta(1-\tau)^2\left(w_t^{Bev,PAYG}\right)^2(h_L^2+h_H^2)}{4[\alpha(1+\beta)+\tau(1-\alpha)]}, \tag{A4}$$

$$L_{t+1}^{Bev,PAYG} = \frac{N_{t+1}}{2} w_{t+1}^{Bev,PAYG}(1-\tau)(h_L^2+h_H^2), \tag{A5}$$

$$\begin{aligned} k_{t+1}^{Bev,PAYG} &= \frac{K_{t+1}^{Bev,PAYG}}{L_{t+1}^{Bev,PAYG}} = \frac{\dfrac{\alpha N_t \beta(1-\tau)^2\left(w_t^{Bev,PAYG}\right)^2(h_L^2+h_H^2)}{4[\alpha(1+\beta)+\tau(1-\alpha)]}}{\dfrac{N_{t+1}}{2} w_{t+1}^{Bev,PAYG}(1-\tau)(h_L^2+h_H^2)} \\ &= \frac{\alpha\beta(1-\tau)\left(w_t^{Bev,PAYG}\right)^2}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho_{t+1})w_{t+1}^{Bev,PAYG}} \\ &= \frac{\alpha\beta(1-\tau)A^2(1-\alpha)^2\left(k_t^{Bev,PAYG}\right)^{2\alpha}}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho_{t+1})A(1-\alpha)\left(k_{t+1}^{Bev,PAYG}\right)^{\alpha}} \\ &= \frac{\alpha\beta(1-\tau)A(1-\alpha)\left(k_t^{Bev,PAYG}\right)^{2\alpha}}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho_{t+1})\left(k_{t+1}^{Bev,PAYG}\right)^{\alpha}}. \end{aligned} \tag{A6}$$

Recursive formula for the capital in efficiency units:

$$k_{t+1}^{Bev,PAYG} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}} \left(k_t^{Bev,PAYG}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{A7}$$

Unique nontrivial steady state solution (when $\rho_t = \rho$):

$$k^{Bev,PAYG} = \left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{A8}$$

$$l_L^{Bev,PAYG} = (1-\tau)A(1-\alpha)\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_L. \tag{A9}$$

$$l_H^{Bev,PAYG} = (1-\tau)A(1-\alpha)\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_H. \tag{A10}$$

$$\frac{K_t^{Bev,PAYG}}{N_t} = \frac{h_L^2+h_H^2}{2}(1-\tau)A(1-\alpha)\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{A11}$$

Bismarckian PAYG System

Using Eq. (14), one obtains

$$\Omega_{t+1}^{Bis,PAYG} = \frac{w_{t+1}^{Bis,PAYG}\left(h_L l_{L,t+1}^{Bis,PAYG} + h_H l_{H,t+1}^{Bis,PAYG}\right)}{w_t^{Bis,PAYG}\left(h_L l_{L,t}^{Bis,PAYG} + h_H l_{H,t}^{Bis,PAYG}\right)} = \frac{w_{t+1}^{Bis,PAYG} L_{t+1}^{Bis,PAYG}}{w_t^{Bis,PAYG}(1+\rho_{t+1}) L_t^{Bis,PAYG}}. \quad \text{(A12)}$$

This, combined with Eq. (21), provides

$$\begin{aligned}
\frac{\Omega_{t+1}^{Bis,PAYG}}{R_{t+1}^{Bis,PAYG}} &= \frac{w_{t+1}^{Bis,PAYG} L_{t+1}^{Bis,PAYG}}{w_t^{Bis,PAYG}(1+\rho_{t+1}) L_t^{Bis,PAYG} R_{t+1}^{Bis,PAYG}} \\
&= \frac{A(1-\alpha)\left(k_{t+1}^{Bis,PAYG}\right)^{\alpha} L_{t+1}^{Bis,PAYG}}{w_t^{Bis,PAYG}(1+\rho_{t+1}) L_t^{Bis,PAYG} A\alpha\left(k_{t+1}^{Bis,PAYG}\right)^{\alpha-1}} \\
&= \frac{(1-\alpha) L_{t+1}^{Bis,PAYG} k_{t+1}^{Bis,PAYG}}{w_t^{Bis,PAYG}(1+\rho_{t+1})\alpha L_t^{Bis,PAYG}}.
\end{aligned} \quad \text{(A13)}$$

Then, using Eq. (8) and (13), one obtains

$$\begin{aligned}
l_{i,t}^{Bis,PAYG} &= (1-\tau) w_t^{Bis,PAYG} h_i + \tau w_t^{Bis,PAYG} h_i (1+\rho_{t+1}) \frac{\Omega_{t+1}^{Bis,PAYG}}{R_{t+1}^{Bis,PAYG}} \\
&= (1-\tau) w_t^{Bis,PAYG} h_i \\
&\quad + \tau w_t^{Bis,PAYG} h_i (1+\rho_{t+1}) \frac{(1-\alpha) L_{t+1}^{Bis,PAYG} k_{t+1}^{Bis,PAYG}}{w_t^{Bis,PAYG}(1+\rho_{t+1})\alpha L_t^{Bis,PAYG}} \\
&= (1-\tau) w_t^{Bis,PAYG} h_i + \frac{(1-\alpha)\tau h_i L_{t+1}^{Bis,PAYG} k_{t+1}^{Bis,PAYG}}{\alpha L_t^{Bis,PAYG}} \\
&= (1-\tau) w_t^{Bis,PAYG} h_i \\
&\quad + \frac{(1-\alpha)\tau h_i k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1}) \frac{\lambda_L h_L l_{L,t+1}^{Bis,PAYG} + \lambda_H h_H l_{H,t+1}^{Bis,PAYG}}{\lambda_L h_L l_{L,t}^{Bis,PAYG} + \lambda_H h_H l_{H,t}^{Bis,PAYG}}.
\end{aligned} \quad \text{(A14)}$$

Hence, when $i = L, H$, respectively, one gets

$$l_{L,t}^{Bis,PAYG} = (1-\tau) w_t^{Bis,PAYG} h_L + \frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1}) \frac{\lambda_L h_L l_{L,t+1}^{Bis,PAYG} + \lambda_H h_H l_{H,t+1}^{Bis,PAYG}}{\lambda_L h_L l_{L,t}^{Bis,PAYG} + \lambda_H h_H l_{H,t}^{Bis,PAYG}}, \quad \text{(A15)}$$

$$l_{H,t}^{Bis,PAYG} = (1-\tau) w_t^{Bis,PAYG} h_H + \frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1}) \frac{\lambda_L h_L l_{L,t+1}^{Bis,PAYG} + \lambda_H h_H l_{H,t+1}^{Bis,PAYG}}{\lambda_L h_L l_{L,t}^{Bis,PAYG} + \lambda_H h_H l_{H,t}^{Bis,PAYG}}. \quad \text{(A16)}$$

Multiplying Eq. (A15) and (A16) by h_H and h_L, respectively, and taking the difference, one obtains

$$l_{L,t}^{Bis,PAYG} h_H - l_{H,t}^{Bis,PAYG} h_L = 0, \quad \text{(A17)}$$

so

$$l_{H,t}^{Bis,PAYG} = \frac{h_H}{h_L} l_{L,t}^{Bis,PAYG}, \tag{A18}$$

$$l_{H,t+1}^{Bis,PAYG} = \frac{h_H}{h_L} l_{L,t+1}^{Bis,PAYG}. \tag{A19}$$

Hence, Eq. (A15) simplifies to

$$\begin{aligned} l_{L,t}^{Bis,PAYG} &= (1-\tau) w_t^{Bis,PAYG} h_L \\ &\quad + \frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1}) \frac{\lambda_L h_L l_{L,t+1}^{Bis,PAYG} + \lambda_H h_H^2/h_L l_{L,t+1}^{Bis,PAYG}}{\lambda_L h_L l_{L,t}^{Bis,PAYG} + \lambda_H h_H^2/h_L l_{L,t}^{Bis,PAYG}} \\ &= (1-\tau) w_t^{Bis,PAYG} h_L + \frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1}) \frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}} \\ &= (1-\alpha)(1-\tau) A h_L \left(k_t^{Bis,PAYG}\right)^{\alpha} + \frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1}) \frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}}. \end{aligned} \tag{A20}$$

Multiplying the two sides of Eq. (A20) by $l_{L,t}^{Bis,PAYG}$, one obtains

$$\begin{aligned} \left(l_{L,t}^{Bis,PAYG}\right)^2 &- (1-\alpha)(1-\tau) A h_L \left(k_t^{Bis,PAYG}\right)^{\alpha} l_{L,t}^{Bis,PAYG} \\ &- \frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1}) l_{L,t+1}^{Bis,PAYG} = 0. \end{aligned} \tag{A21}$$

This is a second-order algebraic equation with discriminant $\Delta > 0$, whose only positive solution is

$$\begin{aligned} & l_{L,t}^{Bis,PAYG} \\ &= \frac{(1-\alpha)(1-\tau) A h_L \left(k_t^{Bis,PAYG}\right)^{\alpha}}{2} \\ &+ \frac{\sqrt{(1-\alpha)^2(1-\tau)^2 A^2 h_L^2 \left(k_t^{Bis,PAYG}\right)^{2\alpha} + \frac{4\tau(1-\alpha)(1+\rho_{t+1}) h_L}{\alpha} k_{t+1}^{Bis,PAYG}\, l_{L,t+1}^{Bis,PAYG}}}{2}. \end{aligned} \tag{A22}$$

In the following, we also determine another recurrence satisfied by the capital in efficiency units. To this aim, first we have to find expressions for the savings $s_{i,t}^{Bis,PAYG}$. Using Eq. (A18) and (14), we can simplify the expression of $\frac{\Omega_{t+1}^{Bis,PAYG}}{R_{t+1}^{Bis,PAYG}}$ and determine the expression of $\frac{p_{i,t+1}^{Bis,PAYG}}{R_{t+1}^{Bis,PAYG}}$ as follows.

$$\frac{\Omega_{t+1}^{Bis,PAYG}}{R_{t+1}^{Bis,PAYG}} = \frac{w_{t+1}^{Bis,PAYG}\left(h_L l_{L,t+1}^{Bis,PAYG} + h_H l_{H,t+1}^{Bis,PAYG}\right)}{w_t^{Bis,PAYG}\left(h_L l_{L,t}^{Bis,PAYG} + h_H l_{H,t}^{Bis,PAYG}\right) A\alpha\left(k_{t+1}^{Bis,PAYG}\right)^{\alpha-1}}$$

$$= \frac{w_{t+1}^{Bis,PAYG} l_{L,t+1}^{Bis,PAYG}}{w_t^{Bis,PAYG} l_{L,t}^{Bis,PAYG} A\alpha\left(k_{t+1}^{Bis,PAYG}\right)^{\alpha-1}}$$

$$= \frac{A(1-\alpha)\left(k_{t+1}^{Bis,PAYG}\right)^{\alpha} l_{L,t+1}^{Bis,PAYG}}{A(1-\alpha)\left(k_t^{Bis,PAYG}\right)^{\alpha} l_{L,t}^{Bis,PAYG} A\alpha\left(k_{t+1}^{Bis,PAYG}\right)^{\alpha-1}} \tag{A23}$$

$$= \frac{k_{t+1}^{Bis,PAYG} l_{L,t+1}^{Bis,PAYG}}{A\alpha\left(k_t^{Bis,PAYG}\right)^{\alpha} l_{L,t}^{Bis,PAYG}}.$$

$$\frac{p_{i,t+1}^{Bis,PAYG}}{R_{t+1}^{Bis,PAYG}} = \tau w_t^{Bis,PAYG}(1+\rho_{t+1}) h_i l_{i,t} \frac{\Omega_{t+1}^{Bis,PAYG}}{R_{t+1}^{Bis,PAYG}} \tag{A24}$$

$$= \tau w_t^{Bis,PAYG}(1+\rho_{t+1}) h_i l_{i,t} \frac{k_{t+1}^{Bis,PAYG} l_{L,t+1}^{Bis,PAYG}}{A\alpha\left(k_t^{Bis,PAYG}\right)^{\alpha} l_{L,t}^{Bis,PAYG}}$$

$$= \tau A(1-\alpha)\left(k_t^{Bis,PAYG}\right)^{\alpha}(1+\rho_{t+1}) h_i l_{i,t} \frac{k_{t+1}^{Bis,PAYG} l_{L,t+1}^{Bis,PAYG}}{A\alpha\left(k_t^{Bis,PAYG}\right)^{\alpha} l_{L,t}^{Bis,PAYG}}$$

$$= \tau(1-\alpha)(1+\rho_{t+1}) h_i l_{i,t} \frac{k_{t+1}^{Bis,PAYG} l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}}.$$

Hence, using Eq. (7) and (A20), one obtains the following expressions for $s_{L,t}^{Bis,PAYG}$ and $s_{H,t}^{Bis,PAYG}$.

$$
\begin{aligned}
s_{L,t}^{Bis,PAYG} = & \frac{\beta(1-\tau)^2A^2(1-\alpha)^2\left(k_t^{Bis,PAYG}\right)^{2\alpha}h_L^2}{1+\beta} \\
& + \frac{\beta(1-\tau)A(1-\alpha)^2\left(k_t^{Bis,PAYG}\right)^{\alpha}h_L^2\tau}{(1+\beta)}\frac{k_{t+1}^{Bis,PAYG}}{\alpha}(1 \\
& + \rho_{t+1})\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}} - \frac{\beta(1-\tau)^2A^2(1-\alpha)^2\left(k_t^{Bis,PAYG}\right)^{2\alpha}h_L^2}{2(1+\beta)} \\
& - \frac{\beta}{2(1+\beta)}\frac{(1-\alpha)^2\tau^2h_L^2\left(k_{t+1}^{Bis,PAYG}\right)^2}{\alpha^2}(1+\rho_{t+1})^2\left(\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}}\right)^2 \\
& - \frac{\beta(1-\tau)A(1-\alpha)^2\left(k_t^{Bis,PAYG}\right)^{\alpha}h_L^2\tau}{(1+\beta)}\frac{k_{t+1}^{Bis,PAYG}}{\alpha}(1 \\
& + \rho_{t+1})\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}} \\
& - \frac{\tau(1-\alpha)(1+\rho_{t+1})h_L l_{L,t}^{Bis,PAYG}}{1+\beta}\frac{k_{t+1}^{Bis,PAYG}}{\alpha}\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}} \\
& = \frac{\beta(1-\tau)^2A^2(1-\alpha)^2\left(k_t^{Bis,PAYG}\right)^{2\alpha}h_L^2}{2(1+\beta)} \\
& - \frac{\beta}{2(1+\beta)}\frac{(1-\alpha)^2\tau^2h_L^2\left(k_{t+1}^{Bis,PAYG}\right)^2}{\alpha^2}(1+\rho_{t+1})^2\left(\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}}\right)^2 \\
& - \frac{\tau(1-\alpha)(1+\rho_{t+1})h_L l_{L,t}^{Bis,PAYG}}{1+\beta}\frac{k_{t+1}^{Bis,PAYG}}{\alpha}\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}},
\end{aligned} \tag{A25}
$$

$$
\begin{aligned}
s_{H,t}^{Bis,PAYG} = & \frac{\beta(1-\tau)^2A^2(1-\alpha)^2\left(k_t^{Bis,PAYG}\right)^{2\alpha}h_H^2}{2(1+\beta)} \\
& - \frac{\beta}{2(1+\beta)}\frac{(1-\alpha)^2\tau^2h_H^2\left(k_{t+1}^{Bis,PAYG}\right)^2}{\alpha^2}(1+\rho_{t+1})^2\left(\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}}\right)^2 \\
& - \frac{\tau(1-\alpha)(1+\rho_{t+1})h_H^2 l_{L,t}^{Bis,PAYG}}{h_L(1+\beta)}\frac{k_{t+1}^{Bis,PAYG}}{\alpha}\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}}.
\end{aligned} \tag{A26}
$$

Combining Eq. (A25) and (A26), one gets

$$K_{t+1}^{Bis,PAYG} = \frac{N_t}{2}\sum_{i=L,H} s_{i,t}^{Bis,PAYG}$$

$$= \frac{N_t}{2(1+\beta)}\Bigg[\beta(1-\tau)^2A^2(1-\alpha)^2\left(k_t^{Bis,PAYG}\right)^{2\alpha}\frac{h_L^2+h_H^2}{2}$$
$$-\frac{\beta}{2}\frac{(1-\alpha)^2\tau^2\left(k_{t+1}^{Bis,PAYG}\right)^2}{\alpha^2}(1+\rho_{t+1})^2\left(\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}}\right)^2(h_L^2+h_H^2) \tag{A27}$$
$$-\tau(1-\alpha)(1+\rho_{t+1})\frac{k_{t+1}^{Bis,PAYG}}{\alpha}l_{L,t+1}^{Bis,PAYG}\frac{h_L^2+h_H^2}{h_L}\Bigg].$$

Then, one obtains

$$L_{t+1}^{Bis,PAYG} = \frac{N_{t+1}}{2}\left(h_L l_{L,t+1}^{Bis,PAYG} + h_H l_{H,t+1}^{Bis,PAYG}\right) = \frac{N_t}{2}(1+\rho_{t+1})\frac{h_L^2+h_H^2}{h_L}l_{L,t+1}^{Bis,PAYG}, \tag{A28}$$

$$k_{t+1}^{Bis,PAYG} = \frac{K_{t+1}^{Bis,PAYG}}{L_{t+1}^{Bis,PAYG}}$$
$$= \frac{1}{2(1+\beta)(1+\rho_{t+1})l_{L,t+1}^{Bis,PAYG}}\Bigg[A^2\beta(1-\alpha)^2(1$$
$$-\tau)^2h_L\left(k_t^{Bis,PAYG}\right)^{2\alpha} \tag{A29}$$
$$-\frac{\beta\tau^2(1-\alpha)^2(1+\rho_{t+1})^2h_L}{\alpha^2}\left(\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}}\right)^2\left(k_{t+1}^{Bis,PAYG}\right)^2$$
$$-\frac{2\tau(1-\alpha)(1+\rho_{t+1})k_{t+1}^{Bis,PAYG}}{\alpha}l_{L,t+1}^{Bis,PAYG}\Bigg].$$

In the following, we also show how one can express $k_{t+1}^{Bis,PAYG}$ as a function of $k_t^{Bis,PAYG}$. Starting from Eq. (A21), one obtains

$$l_{L,t+1}^{Bis,PAYG} = \frac{l_{L,t}^{Bis,PAYG}\left(l_{L,t}^{Bis,PAYG}-(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}\right)}{\frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1})}, \tag{A30}$$

which requires

$$l_{L,t}^{Bis,PAYG} \geq (1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}, \tag{A31}$$

to guarantee the non-negativity of $l_{L,t+1}^{Bis,PAYG}$. Then, using also Eq. (A29), one obtains

$$2(1+\beta)(1+\rho_{t+1})k_{t+1}^{Bis,PAYG}\frac{l_{L,t}^{Bis,PAYG}\left(l_{L,t}^{Bis,PAYG}-(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}\right)}{\frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1})}$$

$$= A^2\beta(1-\alpha)^2(1-\tau)^2 h_L\left(k_t^{Bis,PAYG}\right)^{2\alpha}$$

$$-\frac{\beta\tau^2(1-\alpha)^2(1+\rho_{t+1})^2 h_L}{\alpha^2}\frac{\frac{\left(l_{L,t}^{Bis,PAYG}\right)^2\left(l_{L,t}^{Bis,PAYG}-(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}\right)^2}{\frac{(1-\alpha)^2\tau^2 h_L^2\left(k_{t+1}^{Bis,PAYG}\right)^2}{\alpha^2}(1+\rho_{t+1})^2}}{\left(l_{L,t}^{Bis,PAYG}\right)^2}\left(k_{t+1}^{Bis,PAYG}\right)^2 \tag{A32}$$

$$-\frac{2\tau(1-\alpha)(1+\rho_{t+1})k_{t+1}^{Bis,PAYG}}{\alpha}\frac{l_{L,t}^{Bis,PAYG}\left(l_{L,t}^{Bis,PAYG}-(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}\right)}{\frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1})},$$

hence

$$\frac{2\alpha(1+\beta)l_{L,t}^{Bis,PAYG}\left(l_{L,t}^{Bis,PAYG}-(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}\right)}{(1-\alpha)\tau h_L}$$

$$= A^2\beta(1-\alpha)^2(1-\tau)^2 h_L\left(k_t^{Bis,PAYG}\right)^{2\alpha}$$

$$-\frac{\beta\left(l_{L,t}^{Bis,PAYG}-(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}\right)^2}{h_L} \tag{A33}$$

$$-\frac{2\,l_{L,t}^{Bis,PAYG}\left(l_{L,t}^{Bis,PAYG}-(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}\right)}{h_L}.$$

After some simplifications, this reduces to

$$\frac{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}{(1-\alpha)\tau}=\beta\frac{(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}}{l_{L,t}^{Bis,PAYG}-(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha}}. \tag{A34}$$

Concluding, one can express $l_{L,t}^{Bis,PAYG}$ as a function of $k_t^{Bis,PAYG}$ as follows.

$$l_{L,t}^{Bis,PAYG}=\frac{[\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1-\alpha)(1-\tau)A\,h_L}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}\left(k_t^{Bis,PAYG}\right)^{\alpha}, \tag{A35}$$

which also satisfies Eq. (A31). Then, using Eq. (A18), one obtains

$$l_{H,t}^{Bis,PAYG}=\frac{[\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1-\alpha)(1-\tau)A\,h_H}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}\left(k_t^{Bis,PAYG}\right)^{\alpha}. \tag{A36}$$

From Eq. (A35), one gets

$$\frac{l_{L,t+1}^{Bis,PAYG}}{l_{L,t}^{Bis,PAYG}}=\frac{\left(k_{t+1}^{Bis,PAYG}\right)^{\alpha}}{\left(k_t^{Bis,PAYG}\right)^{\alpha}}, \tag{A37}$$

which, combined with (A20) and (A35), provides

$$\frac{[\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1-\alpha)(1-\tau)A\,h_L}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}\left(k_t^{Bis,PAYG}\right)^{\alpha}$$
$$=(1-\alpha)(1-\tau)Ah_L\left(k_t^{Bis,PAYG}\right)^{\alpha} \qquad (A38)$$
$$+\frac{(1-\alpha)\tau h_L k_{t+1}^{Bis,PAYG}}{\alpha}(1+\rho_{t+1})\frac{\left(k_{t+1}^{Bis,PAYG}\right)^{\alpha}}{\left(k_t^{Bis,PAYG}\right)^{\alpha}}.$$

Then, after some simplifications, one obtains

$$k_{t+1}^{Bis,PAYG}=\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)(1+\rho_{t+1})]}\right]^{\frac{1}{1+\alpha}}\left(k_t^{Bis,PAYG}\right)^{\frac{2\alpha}{1+\alpha}}. \qquad (A39)$$

Summarizing the analysis above, one obtains the following for the Bismarckian PAYG case.

Relationship between the labor supply choices of the low-skilled and high-skilled individuals:

$$l_{H,t}^{Bis,PAYG}=\frac{h_H}{h_L}l_{L,t}^{Bis,PAYG}. \qquad (A40)$$

Recursive formula for the capital in efficiency units:

$$k_{t+1}^{Bis,PAYG}=\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}}\left(k_t^{Bis,PAYG}\right)^{\frac{2\alpha}{1+\alpha}}. \qquad (A41)$$

Unique nontrivial steady state solution (when $\rho_t=\rho$):

$$k^{Bis,PAYG}=\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{1}{1-\alpha}}. \qquad (A42)$$

$$l_L^{Bis,PAYG}=(1-\tau)A(1-\alpha)\frac{\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}$$
$$\times\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}}h_L. \qquad (A43)$$

$$l_H^{Bis,PAYG}=(1-\tau)A(1-\alpha)\frac{\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}$$
$$\times\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}}h_H. \qquad (A44)$$

$$\frac{K_t^{Bev,PAYG}}{N_t}=\frac{h_L^2+h_H^2}{2}(1-\tau)A(1-\alpha)\frac{\beta\tau(1-\alpha)+2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)}$$
$$\times\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{1}{1-\alpha}}. \qquad (A45)$$

Beveridgean FF System

Combining Eq. (8) and (16), and taking into account that in a Beveridgean payment scheme - the labor supply decisions of the other individuals being the same - the labor supply decision of a single individual does not practically influence his future pension, one obtains

$$l_{i,t}^{Bev,FF} = (1-\tau) w_t^{Bev,FF} h_i. \tag{A46}$$

This, combined with Eq. (7) and (16) again, provides

$$s_{i,t}^{Bev,FF} = \frac{\beta(1-\tau) w_t^{Bev,FF} h_i l_{i,t}^{Bev,FF} - \frac{\beta\left(l_{i,t}^{Bev,FF}\right)^2}{2} - \frac{R_{t+1}^{Bev,FF} \tau w_t^{Bev,FF}\left(h_L l_{L,t}^{Bev,FF} + h_H l_{H,t}^{Bev,FF}\right)}{2R_{t+1}^{Bev,FF}}}{1+\beta}. \tag{A47}$$

Then, using Eq. (25), (22), and (23), one gets

$$\begin{aligned} K_{t+1}^{Bev,FF} &= \frac{N_t}{2} \sum_{i=L,H} \left(s_{i,t}^{Bev,FF} + \tau w_t^{Bev,FF} h_i l_{i,t}^{Bev,FF}\right) \\ &= \frac{N_t}{2}\Bigg[\frac{\beta(1-\tau) w_t^{Bev,FF} h_L l_{L,t}^{Bev,FF}}{1+\beta} + \frac{\beta(1-\tau) w_t^{Bev,FF} h_H l_{H,t}^{Bev,FF}}{1+\beta} \\ &- \frac{\tau w_t^{Bev,FF}\left(h_L l_{L,t}^{Bev,FF} + h_H l_{H,t}^{Bev,FF}\right)}{1+\beta} - \frac{\beta\left(l_{L,t}^{Bev,FF}\right)^2}{2(1+\beta)} - \frac{\beta\left(l_{H,t}^{Bev,FF}\right)^2}{2(1+\beta)} \\ &+ \frac{(1+\beta)\tau w_t^{Bev,FF}\left(h_L l_{L,t}^{Bev,FF} + h_H l_{H,t}^{Bev,FF}\right)}{1+\beta}\Bigg] \\ &= \frac{\beta N_t w_t^{Bev,FF}\left(h_L l_{L,t}^{Bev,FF} + h_H l_{H,t}^{Bev,FF}\right)}{2(1+\beta)} \\ &- \frac{\beta N_t\left(\left(l_{L,t}^{Bev,FF}\right)^2 + \left(l_{H,t}^{Bev,FF}\right)^2\right)}{4(1+\beta)}, \end{aligned} \tag{A48}$$

$$L_{t+1}^{Bev,FF} = \frac{N_{t+1}}{2} w_{t+1}^{Bev,FF}(1-\tau)(h_L^2 + h_H^2), \tag{A49}$$

$$
\begin{aligned}
k_{t+1}^{Bev,FF} &= \frac{K_{t+1}^{Bev,FF}}{L_{t+1}^{Bev,FF}} \\
&= \frac{\frac{\beta N_t w_t^{Bev,FF}\left(h_L l_{L,t}^{Bev,FF} + h_H l_{H,t}^{Bev,FF}\right)}{2(1+\beta)}}{\frac{N_{t+1}}{2}\, w_{t+1}^{Bev,FF}(1-\tau)(h_L^2+h_H^2)} \\
&- \frac{\frac{\beta N_t\left(\left(l_{L,t}^{Bev,FF}\right)^2 + \left(l_{H,t}^{Bev,FF}\right)^2\right)}{4(1+\beta)}}{\frac{N_{t+1}}{2} w_{t+1}^{Bev,FF}(1-\tau)(h_L^2+h_H^2)} \\
&= \frac{\frac{\beta N_t(1-\tau)\left(w_t^{Bev,FF}\right)^2(h_L^2+h_H^2)}{2(1+\beta)}}{\frac{N_{t+1}}{2}\, w_{t+1}^{Bev,FF}(1-\tau)(h_L^2+h_H^2)} \\
&- \frac{\frac{\beta N_t(1-\tau)^2\left(w_t^{Bev,FF}\right)^2(h_L^2+h_H^2)}{4(1+\beta)}}{\frac{N_{t+1}}{2} w_{t+1}^{Bev,FF}(1-\tau)(h_L^2+h_H^2)} \\
&= \frac{\beta\left(w_t^{Bev,FF}\right)^2}{(1+\beta)(1+\rho_{t+1})w_{t+1}^{Bev,FF}} - \frac{\beta(1-\tau)\left(w_t^{Bev,FF}\right)^2}{2(1+\beta)(1+\rho_{t+1})w_{t+1}^{Bev,FF}} \\
&= \frac{\beta(1+\tau)\left(w_t^{Bev,FF}\right)^2}{2(1+\beta)(1+\rho_{t+1})w_{t+1}^{Bev,FF}} \\
&= \frac{\beta(1+\tau)A^2(1-\alpha)^2\left(k_t^{Bev,FF}\right)^{2\alpha}}{2(1+\beta)(1+\rho_{t+1})A(1-\alpha)\left(k_{t+1}^{Bev,FF}\right)^{\alpha}} \\
&= \frac{\beta(1+\tau)A(1-\alpha)\left(k_t^{Bev,FF}\right)^{2\alpha}}{2(1+\beta)(1+\rho_{t+1})\left(k_{t+1}^{Bev,FF}\right)^{\alpha}}.
\end{aligned}
\tag{A50}
$$

Recursive formula for the capital in efficiency units:

$$
k_{t+1}^{Bev,FF} = \left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}} \left(k_t^{Bev,FF}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{A51}
$$

Unique nontrivial steady state solution (when $\rho_t = \rho$):

$$
k^{Bev,FF} = \left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{A52}
$$

$$
l_L^{Bev,FF} = (1-\tau)A(1-\alpha)\left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_L. \tag{A53}
$$

$$l_H^{Bev,FF} = (1-\tau)A(1-\alpha)\left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_H. \tag{A54}$$

$$\frac{K_t^{Bev,FF}}{N_t} = \frac{h_L^2 + h_H^2}{2}(1-\tau)A(1-\alpha)\left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{A55}$$

Bismarckian FF Pension System

Combining Eq. (8) and (17), one obtains

$$l_{i,t}^{Bis,FF} = w_t^{Bis,FF} h_i. \tag{A56}$$

This, combined with Eq. (7) and (17) again, provides

$$\begin{aligned} s_{i,t}^{Bis,FF} &= \frac{\beta(1-\tau)w_t^{Bis,FF} h_i l_{i,t}^{Bis,FF} - \frac{\beta\left(l_{i,t}^{Bis,FF}\right)^2}{2} - \frac{R_{t+1}^{Bis,FF}\tau w_t^{Bis,FF} h_i l_{i,t}^{Bis,FF}}{R_{t+1}^{Bis,FF}}}{1+\beta} \\ &= \frac{\beta(1-\tau)\left(w_t^{Bis,FF}\right)^2 h_i^2}{1+\beta} - \frac{\beta\left(w_t^{Bis,FF}\right)^2 h_i^2}{2(1+\beta)} - \frac{\tau\left(w_t^{Bis,FF}\right)^2 h_i^2}{1+\beta} \\ &= \frac{(\beta - 2\tau\beta - 2\tau)\left(w_t^{Bis,FF}\right)^2 h_i^2}{2(1+\beta)}. \end{aligned} \tag{A57}$$

Then, using Eq. (25), (22), and (23), one gets

$$\begin{aligned} K_{t+1}^{Bis,FF} &= \frac{N_t}{2}\sum_{i=L,H}\left(s_{i,t}^{Bis,FF} + \tau w_t^{Bis,FF} h_i l_{i,t}^{Bis,FF}\right) \\ &= \frac{N_t}{2}\left[\frac{(\beta - 2\tau\beta - 2\tau)\left(w_t^{Bis,FF}\right)^2 (h_L^2 + h_H^2)}{2(1+\beta)}\right. \\ &\quad \left. + \frac{2(1+\beta)\tau\left(w_t^{Bis,FF}\right)^2 (h_L^2 + h_H^2)}{2(1+\beta)}\right] = \frac{N_t\beta\left(w_t^{Bis,FF}\right)^2 (h_L^2 + h_H^2)}{4(1+\beta)}, \end{aligned} \tag{A58}$$

$$L_{t+1}^{Bis,FF} = \frac{N_{t+1}}{2} w_{t+1}^{Bis,FF}(h_L^2 + h_H^2), \tag{A59}$$

$$k_{t+1}^{Bis,FF}=\frac{K_{t+1}^{Bis,FF}}{L_{t+1}^{Bis,FF}}=\frac{\frac{N_t\beta\left(w_t^{Bis,FF}\right)^2(h_L^2+h_H^2)}{4(1+\beta)}}{\frac{N_{t+1}}{2}w_{t+1}^{Bis,FF}(h_L^2+h_H^2)}=\frac{\beta\left(w_t^{Bis,FF}\right)^2}{2(1+\beta)(1+\rho_{t+1})w_{t+1}^{Bis,FF}}$$

$$=\frac{\beta A^2(1-\alpha)^2\left(k_t^{Bis,FF}\right)^{2\alpha}}{2(1+\beta)(1+\rho_{t+1})A(1-\alpha)\left(k_{t+1}^{Bis,FF}\right)^{\alpha}} \tag{A60}$$

$$=\frac{\beta A(1-\alpha)\left(k_t^{Bis,FF}\right)^{2\alpha}}{2(1+\beta)(1+\rho_{t+1})\left(k_{t+1}^{Bis,FF}\right)^{\alpha}}$$

$$=\frac{\alpha\beta(1-\tau)A(1-\alpha)\left(k_t^{Bev,PAYG}\right)^{2\alpha}}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho_{t+1})\left(k_{t+1}^{Bev,PAYG}\right)^{\alpha}}.$$

Recursive formula for the capital in efficiency units:

$$k_{t+1}^{Bis,FF}=\left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho_{t+1})}\right]^{\frac{1}{1+\alpha}}\left(k_t^{Bis,FF}\right)^{\frac{2\alpha}{1+\alpha}}. \tag{A61}$$

Unique nontrivial steady state solution (when $\rho_t=\rho$):

$$k^{Bis,FF}=\left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{A62}$$

$$l_L^{Bis,FF}=A(1-\alpha)\left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}}h_L. \tag{A63}$$

$$l_H^{Bis,FF}=A(1-\alpha)\left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}}h_H. \tag{A64}$$

$$\frac{K_t^{Bis,FF}}{N_t}=\frac{h_L^2+h_H^2}{2}A(1-\alpha)\left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{\frac{1}{1-\alpha}}. \tag{A65}$$

Stability analysis

All the difference equations (A7), (A41), (A51), (A61) describing the evolution of k_t for the four pensions systems are of the form

$$k_{t+1}=Ck_t^{\frac{2\alpha}{1+\alpha}}, \tag{A66}$$

where $C>0$ is a constant which depends on the pension system. As it is usual in the analysis of one-dimensional autonomous time-invariant dynamical systems (see, e.g., de la Croix and Michel

(2002), Appendix A.3), existence and local stability of steady state solutions can be investigated graphically, finding the intersections between the curve

$$k_{t+1}(k_t) = Ck_t^{\frac{2\alpha}{1+\alpha}}, \tag{A67}$$

and the line

$$k_{t+1}(k_t) = k_t, \tag{A68}$$

and evaluating the local slope of the first curve at such intersections (see Figure A1, which refers to a particular choice of the parameters, which does not influence the result of the stability analysis).

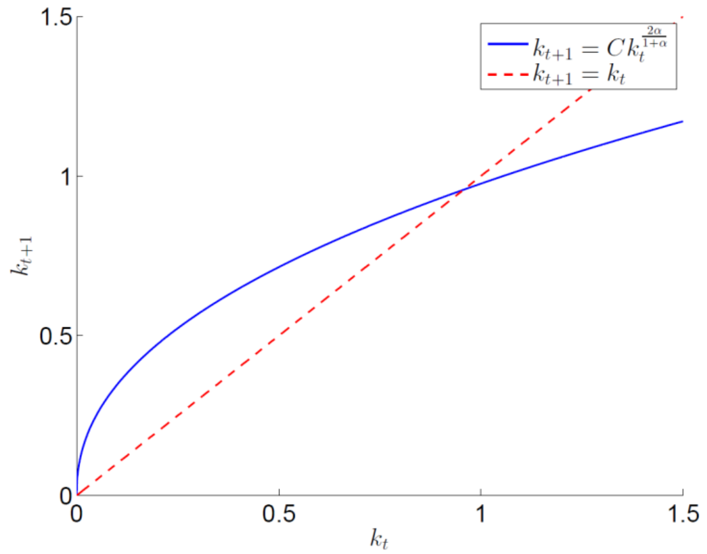

Figure A1. For $\alpha = 0.1$ and $C = 0.9763$: graphical investigation of the existence of steady states for the dynamical system $k_{t+1} = Ck_t^{\frac{2\alpha}{1+\alpha}}$, and of their local stability.

Hence, one can see from Figure A1 that such difference equations admit the trivial steady state $\bar{k}^{(1)} = 0$, and the nontrivial one $\bar{k}^{(2)} = C^{\frac{1+\alpha}{1-\alpha}}$. Since the slope of the first curve is infinite in the trivial steady state $\bar{k}^{(1)}$ and less than 1 in absolute value in the nontrivial steady state $\bar{k}^{(2)}$, we conclude that the trivial steady state $\bar{k}^{(1)}$ is unstable, whereas the nontrivial one $\bar{k}^{(2)}$ is locally asymptotically stable. Moreover, using a similar analysis as in de la Croix and Michel (2002), Appendix A.3 (see, in particular, Proposition A.6 in that reference and its proof), one can also prove that $\bar{k}^{(2)}$ is even globally asymptotically stable (i.e., for any initial condition different from 0, k_t

tends to $\bar{k}^{(2)}$ as t tends to infinity). This follows from the fact that the right-hand side of each difference equation $k_{t+1} = Ck_t^{\frac{2\alpha}{1+\alpha}}$ is a non-negative, concave, and increasing function of k_t.

Partial derivatives with respect to τ and ρ of the steady state values of the capital in efficiency units and of the individual labor supply choice

In the following, we report closed-form expressions for the partial derivatives mentioned in Propositions 1, 2, and 3, together with their signs.

$$\frac{\partial k^{Bev,PAYG}}{\partial \tau} = -\frac{\frac{A\alpha\beta}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)} + \frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)]^2(1+\rho)}}{\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{-\frac{1}{1-\alpha}+1}} < 0. \tag{A69}$$

$$\frac{\partial k^{Bis,PAYG}}{\partial \tau} == -\frac{\frac{A\alpha\beta}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)} + \frac{A\alpha\beta(1-\alpha)(2+\beta)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)]^2(1+\rho)}}{\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{-\frac{1}{1-\alpha}+1}} < 0. \tag{A70}$$

$$\frac{\partial k^{Bev,FF}}{\partial \tau} = \frac{A\beta}{2(1+\beta)(1+\rho)\left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{-\frac{1}{1-\alpha}+1}} > 0. \tag{A71}$$

$$\frac{\partial k^{Bis,FF}}{\partial \tau} = 0. \tag{A72}$$

$$\frac{\partial k^{Bev,PAYG}}{\partial \rho} = -\frac{A\alpha\beta(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)^2\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{-\frac{1}{1-\alpha}+1}} < 0. \tag{A73}$$

$$\frac{\partial k^{Bis,PAYG}}{\partial \rho} = -\frac{A\alpha\beta(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)^2\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{-\frac{1}{1-\alpha}+1}} < 0. \tag{A74}$$

$$\frac{\partial k^{Bev,FF}}{\partial \rho} = -\frac{A\beta(1+\tau)}{2(1+\beta)(1+\rho)^2\left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{-\frac{1}{1-\alpha}+1}} < 0. \tag{A75}$$

$$\frac{\partial k^{Bis,FF}}{\partial \rho} = -\frac{A\beta}{2(1+\beta)(1+\rho)^2\left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{-\frac{1}{1-\alpha}+1}} < 0. \tag{A76}$$

$$\frac{\partial l_i^{Bev,PAYG}}{\partial \tau} = -A(1-\alpha)\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}} h_i - \frac{A\alpha\left[\frac{A\alpha\beta(1-\alpha)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}+\frac{A\alpha\beta(1-\alpha)^2(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)]^2(1+\rho)}\right](1-\tau)}{\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{-\frac{1}{1-\alpha}+2}} h_i < 0. \tag{A77}$$

$$\begin{aligned}\frac{\partial l_i^{Bis,PAYG}}{\partial \tau} = &-\frac{\frac{A^2\alpha^2\beta(1-\alpha)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}(1-\tau)[\tau(1-\alpha)(2+\beta)+2\alpha(1+\beta)+\beta\tau(1-\alpha)]}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)]\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{-\frac{1}{1-\alpha}+2}} h_i \\
&-\frac{\frac{A^2\alpha^2\beta(1-\alpha)^2(2+\beta)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)]^2(1+\rho)}(1-\tau)[\tau(1-\alpha)(2+\beta)+2\alpha(1+\beta)+\beta\tau(1-\alpha)]}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)]\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{-\frac{1}{1-\alpha}+2}} h_i \\
&+\frac{A\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)[\beta(1-\alpha)+(1-\alpha)(2+\beta)](1-\tau)}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)} h_i \\
&-\frac{A\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)^2(2+\beta)(1-\tau)[\tau(1-\alpha)(2+\beta)+2\alpha(1+\beta)+\beta\tau(1-\alpha)]}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)]^2} h_i \\
&-\frac{A\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)[\tau(1-\alpha)(2+\beta)+2\alpha(1+\beta)+\beta\tau(1-\alpha)]}{2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)} h_i\end{aligned} \tag{A78}$$

(the sign is parameter dependent).

$$\frac{\partial l_i^{Bev,FF}}{\partial \tau} = -A\left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)h_i + \frac{A^2\alpha\beta(1-\alpha)(1-\tau)}{2(1+\beta)(1+\rho)\left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{-\frac{1}{1-\alpha}+2}} h_i \tag{A79}$$

(the sign is parameter dependent).

$$\frac{\partial l_i^{Bis,FF}}{\partial \tau} = 0. \tag{A80}$$

$$\frac{\partial l_i^{Bev,PAYG}}{\partial \rho} = -\frac{A^2\alpha^2\beta(1-\alpha)(1-\tau)^2}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)^2\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{2[\alpha(1+\beta)+\tau(1-\alpha)](1+\rho)}\right]^{-\frac{1}{1-\alpha}+2}} h_i < 0. \tag{A81}$$

$$\frac{\partial l_i^{Bis,PAYG}}{\partial \rho} = -\frac{A^2\alpha^2\beta(1-\alpha)(1-\tau)^2[\tau(1-\alpha)(2+\beta)+2\alpha(1+\beta)+\beta\tau(1-\alpha)]}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)]^2(1+\rho)^2\left[\frac{A\alpha\beta(1-\alpha)(1-\tau)}{[2\alpha(1+\beta)+\tau(1-\alpha)(2+\beta)](1+\rho)}\right]^{-\frac{1}{1-\alpha}+2}} h_i < 0. \tag{A82}$$

$$\frac{\partial l_i^{Bev,FF}}{\partial \rho} = -\frac{A^2\alpha\beta(1-\alpha)(1-\tau)(1+\tau)}{2(1+\beta)(1+\rho)^2\left[\frac{A\beta(1-\alpha)(1+\tau)}{2(1+\beta)(1+\rho)}\right]^{-\frac{1}{1-\alpha}+2}} h_i < 0. \tag{A83}$$

$$\frac{\partial l_i^{Bis,FF}}{\partial \rho} = -\frac{A^2\alpha\beta(1-\alpha)}{2(1+\beta)(1+\rho)^2\left[\frac{A\beta(1-\alpha)}{2(1+\beta)(1+\rho)}\right]^{-\frac{1}{1-\alpha}+2}} h_i < 0. \tag{A84}$$