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Research Area
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# The potential costs of Longevity Risk on Public Pensions. Evidence from Italian data.

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**Abstract**

In this article, we assess, through an empirical investigation based on Italian data, how uncertainty regarding future mortality may affect public pension expenditure. Based on a representative sample of Italian pensioners from 1985 to 2011, we find a consistent underestimation of improvements seen in mortality and life expectancy when forecasts are based on expectations. The pension expenditure estimated using realized mortality rates is shown to be consistently higher than that obtained by using average forecasted scenarios, produced with well-known stochastic mortality models. The paper highlights the importance of considering the uncertainty regarding future pension benfits, i.e. of evaluating and managing the longevity risk in public pension plans.

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**Keywords:** longevity risk, mortality model, pension, retirement
**JEL Classification:** C15, C32, J11, J26

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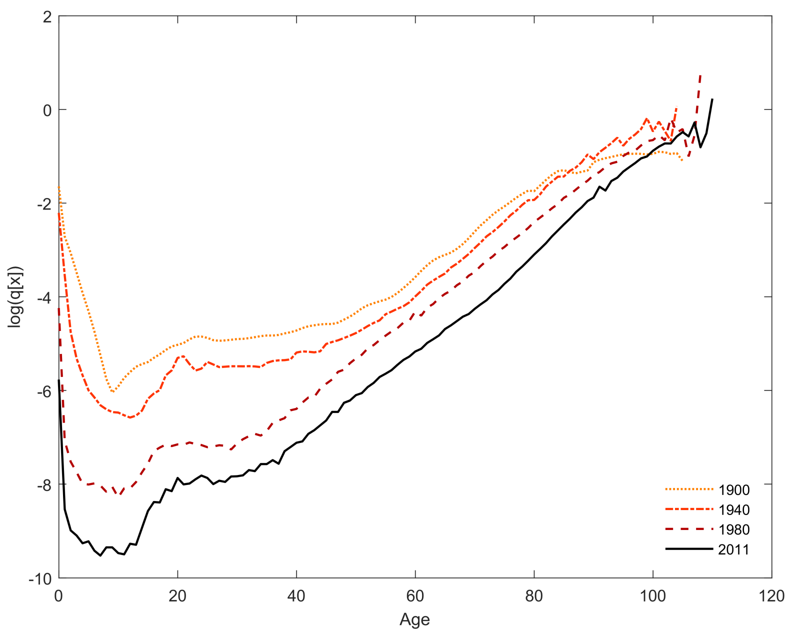
## 1. Introduction

In all industrialized countries, mortality rates experienced a sharp decline during the last century (see for instance OECD (2014)). The determinants of such decline were a decrease in infant mortality and in mortality at older ages, caused by progresses in medicine and improvements in food and living habits. This increasing longevity trend has not slowed down recently, as confirmed by recent analyses (OECD (2016)). In OECD countries, life expectancy at 65 increased from 15.8 years in 2001 to 17.6 years in 2011 for males and from 19.4 to 21 years for females. The Italian experience is in line with these observations. As Figure 1 depicts, mortality rates in Italy fell dramatically over the period 1900-2011. In the period 2001-2011, for instance, life expectancy at age 65 increased by 1.5 years for males and 1.2 years for females (OECD (2016)).

**Figure 1:** Italian observed log mortality rates 1900-2011




Source: HMD (2015)

Most importantly, not only life expectancy has increased constantly in the last century, but this rise has exceeded all expectations (IMF (2012)). No matter the approach chosen to provide forecasts, future mortality rates seem to have been constantly under-estimated by models.[1] Such discrepancy between actual and expected life spans, and its related uncertainty, is called longevity risk. Indeed, while there are many positive aspects are related to the increase in the life expectancy of individuals, there are some, at present highly debated, economic issues due to longevity risk. This risk, that is a trend risk, is relevant for individuals, companies and governments. One consequence it bears is the unexpected increase in public expenditures for the old-age, such as pensions and health expenditure. On the one hand, individuals run the risk of outliving their resources and being forced to reduce their standard of living at old ages. On the other hand, public pension schemes, as well as insurance companies and private pension funds, run the risk of paying out more than they expected to, because of unexpectedly longer life times of their insureds.

In this paper, we focus on the relevance of longevity risk for public pension providers. Indeed, they are affected by longevity risk, because they need to rely on forecasts about the future evolution of mortality rates, on which their calculations of pension benefits should be based, as highlighted by Bisetti and Favero (2014). While they are able to cope easily with the idiosyncratic, random variation risk by pooling different individuals and relying on the law of large numbers, they seem to have no strategy to defend themselves against this systematic risk. The Ageing Report of the EC (2015) stresses how unexpected longevity gains may cause severe financial troubles, in countries where the demographic trends projected over the long term reveal a remarkable ageing process. Actually, in those countries where the old-age dependency ratio[2] is projected to increase in the future, the underestimation of mortality improvements has already exacerbated the financial burden for governments providing public pensions (generally financed via a pay-as-you-go system), forcing them to pay for retirement benefits even more than expected.[3]

In this article, we evaluate how uncertainty regarding future mortality and life ex-

[1]Different actuarial methods can be used to obtain mortality projections (see Booth and Tickle (2008), and Pitacco et al. (2009)). Extrapolative methods, which can be either deterministic or stochastic depending on weather forecasts are built by extending past trends or by using probability distributions, make use of historical data to forecast future mortality rates, i.e. they assume that future trends will essentially be a continuation of the past. Conversely, econometric models investigate drivers of mortality (e.g. economic, environmental and epidemiological factors) in order to produce forecasts. Moreover, forecasting mortality can be based on subjective opinions of experts.

[2]The old age dependency ratio is the ratio of older dependents (people older than 64) to the working-age population (those aged 15-64).

[3]According to the EC (2015), EU countries are facing a significant aging problem. As reported by Eurostat (2013) life expectancy at birth in 2060 will reach 84.8 years for males and 89.1 years for females. Such aging process leads to a substantial increase in the old-age dependency ratio, which is projected to raise from 27.8% to 50.1% over the period 2013-2060.

pectancy outcomes may affect a public pension budget. We do this through an empirical analysis, based on a dataset of Italian pensioners. Using data from WHIP (2015) - Work Histories Italian Panel, about the first state pension amounts paid to 43,641 Italians who retired between 1985-2004, we estimate the pension expenditure borne by the Italian government to pay their retirement benefits in the period 1985-2011, making use of observed mortality rates from the HMD (2015) - Human Mortality Database. Afterwards, we compare such estimation with the forecasted pension expenditure for the same sample of individuals, obtained using forecasted mortality rates. These forecasts are obtained using two well-known stochastic mortality models: the Lee and Carter (1992) and the Cairns et al. (2006) model.[4]

In particular, we first compare the actual pension expenditure with the one produced by applying the forecasts obtained using the Lee and Carter (LC from now on) model, and the historical information avilable in the base year 1984, at the beginning of the sample period. Then, as a stress, we compare the real pension expenditure with the one obtained by updating the LC forecasts every year. Finally, we compare the actual pension expenditure with the one achieved by applying the Cairns et al. model (CBD from now on) in the base year 1984.[5]

We find a consistent underestimation of improvements seen in mortality and life expectancy, when forecasts are based on expectations. The pension expenditure computed by taking into account observed mortality rates is 2.30% higher than the expenditure borne when an average scenario predicted by the LC model in the base year 1984 is used, implying that people have lived longer than expected. However, when yearly updates of the Lee and Carter forecasts are employed, the underestimation reduces to 0.61%. Finally, the application of the average scenario of the CBD model leads to an underestimation of the real pension expenditure of 1.52%, one third lower than what we obtained with the Lee and Carter model. Applying the CBD model widens also the variability around the central estimates, relative to the LC model.

The paper proceeds as follows. Section 2 quantifies the pension expenditure when observed mortality rates and forecasted mortality rates based on Lee and Carter model on the base year 1984 are used. Section 3 discusses robustness by applying yearly updates of the Lee and Carter estimates and the Cairns et al. model to relevant ages. Finally, Section 4 provides some comments and concludes.

[4]For advantages and drawbacks of these two models, see for instance OECD (2014).

[5]We apply the Cairns et al. model in the base year 1984 to forecast mortality rates for the age range 60-90. Mortality rates for the ages 0-59 and 91-110 are those obtained with the Lee and Carter model in the base year 1984.

## 2. Estimating the impact of longevity risk on the Italian pension expenditure

The objective of this paper is to measure how uncertainty regarding future mortality and life expectancy outcomes might affect a public state pension budget when paying for retirement benefits. To this end, we focus on the Italian case. We try to estimate the actual individual retirement benefits paid by the state to a sample of retirees and compare it with their forecasts at inception, according to standard stochastic mortality models. We resort to the WHIP (2015) - Work Histories Italian Panel database to have access to individual data on pensions paid to a sample of retirees. The WHIP dataset on pensions[6] covers the time span 1985-2004 with a representative 1:180 sample, keeping track of 43,641 individuals. Table 1 summarizes the composition of the sample by year of retirement, showing that the number of retirees is between 1400 and 3000 for each year.

Unfortunately, the database does not provide us with the history of the individuals in retirement, but only with the initial amount of their pension. Nonetheless, the database provides us with personal information on the retirees, allowing us to aggregate them by year of birth. In Section 2.1, we estimate the pension expenditure borne by the State for the whole sample of individuals, using the realized mortality rates of the total Italian population to reproduce the pattern of deaths in our sample, which we consider as representative of the whole retirees' population. In Section 2.2 we estimate the pension expenditure according to actuarial projections regarding the development of mortality rates, based on the data available at the beginning of our observation period, 1984.

### *2.1. Estimating the actual pension expenditure*

To provide an estimate of the actual pension expenditure for the retirees in our sample, we employ the following procedure. For each individual in our sample, retired between 1985 and 2004, we project the initial pension amount until 2011, adjusting such an amount for the annual average inflation rate.[7] We then aggregate the individual amounts by year of birth of the retirees. As a result, we obtain the pension expenditure up to 2011 relative to the cohorts of individuals born between 1902-1984. Then, we consider the pattern of deaths in our sample, by applying to each cohort the observed survival rates provided by the HMD (2015) for the Italian population. We multiply the inflated cohort-based pension expenditures by these survival rates and obtain an estimate of the total pension expenditure for our sample up to 2011, aggregating across

[6]The types of retirement benefit paid are: *Dipendenti, Autonomi, Agricoli, Altre gestioni, Pensioni sociali, Trattamenti diretti, Rendite Inail, Superstiti.*

[7]See www.inflation.eu.

**Table 1:** Number of pensioners by year of retirement

| Year of Retirement | Number of pensioners |
|---|---|
| 1985 | 1401 |
| 1986 | 1592 |
| 1987 | 1822 |
| 1988 | 1958 |
| 1989 | 2002 |
| 1990 | 2049 |
| 1991 | 2262 |
| 1992 | 2884 |
| 1993 | 1878 |
| 1994 | 2754 |
| 1995 | 2295 |
| 1996 | 2965 |
| 1997 | 2591 |
| 1998 | 2288 |
| 1999 | 2570 |
| 2000 | 2154 |
| 2001 | 2200 |
| 2002 | 1927 |
| 2003 | 1984 |
| 2004 | 2065 |
| Total | 43,641 |

| | Year of retirement | | | |
|---|---|---|---|---|
| | 1985-1989 | 1990-1994 | 1995-1999 | 2000-2004 |
| Number of pensioneers | 8775 | 11,827 | 12,709 | 10,330 |

Source: WHIP (2015)

cohorts and years (1985-2011). Our estimate for the 43,641 sample individuals amounts to 6,354,610,346 Euros.

Table 2 disentangles the pension expenditure by year of retirement. The first years, even if more distant from the end of our observation period, have a relatively low expenditure due to the smaller number of individuals in the sample.

### *2.2. The forecasted pension expenditure: mortality rates from the Lee and Carter (1992) model*

We now compare our estimates of the actual pension benefits paid to our sample of retirees with a set of forecasts, based on actuarial models. These forecasts are relevant, since the pension amount granted to an individual as a public pension is not subject to revision, if the survivorship unexpectedly changes. Nonetheless, actuarial fairness would encourage to fix such pension amount appropriately, based on as-accurate-as-possible estimates of the residual lifetimes. Discrepancies between expected and realized mortality rates are exactly what we refer to as longevity risk, and determine the unexpected excess payment to pensioners in the case of improved longevity relative to forecasts.

**Table 2:** Pension expenditure (in Euro) for sample retirees in the period 1985-2011 (by year of retirement 1985-2004)

| Year of Retirement | Expenditure |
|---|---|
| 1985 | 200,969,315 |
| 1986 | 234,512,955 |
| 1987 | 267,027,489 |
| 1988 | 294,324,699 |
| 1989 | 303,963,558 |
| 1990 | 328,435,410 |
| 1991 | 351,769,440 |
| 1992 | 522,936,015 |
| 1993 | 267,478,388 |
| 1994 | 508,229,840 |
| 1995 | 378,347,963 |
| 1996 | 466,544,569 |
| 1997 | 418,508,243 |
| 1998 | 313,819,978 |
| 1999 | 331,606,150 |
| 2000 | 252,950,271 |
| 2001 | 264,320,748 |
| 2002 | 227,568,902 |
| 2003 | 211,640,947 |
| 2004 | 209,655,464 |
| Total | 6,354,610,346 |

Source: Authors' calculation

First, we produce forecasts based on 1984 data, using the most well-known stochastic mortality model, the LC one.[8]

The LC model describes the age-period surface of log mortality rates $log(m_{xt})$ as:

$$log(m_{xt}) = a_x + b_x k_t + \epsilon_{x,t}. \tag{1}$$

The vector $a_x$ can be interpreted as an average age profile, the vector $k_t$ tracks mortality changes over time, the vector $b_x$ determines how much each age group changes when $k_t$ changes, and the error term $\epsilon_{x,t}$, reflects particular age-specific historical influences not captured in the model. It is well-known that the above model is over-parametrized and, thus, constraints need to be imposed in order to identify it. Following the standard identification procedure, we impose $b_x$'s to sum to one and $k_t$'s to sum to zero. As a consequence of this particular choice, $a_x$'s are the average log rates.

[8]Obviously, in 1984, the LC model had not yet been formulated, but our choice of using the LC model is justified by the fact that the Italian Statistical Institute, ISTAT, now makes use of such a model to forecast age-specific mortality rates.

By using Italian mortality data from 1935 to 1984,[9] we estimate $a_x$ averaging log-rates over time and $b_x$ and $k_t$ via singular value decomposition of the residuals. This method approximates a matrix as the product of two vectors.

However, as argued in Lee and Carter (1992), the fitted death rates derived in this way generally will not lead to the actual numbers of deaths when applied to given population age distribution. For this reason, by making use of Equation 1, we apply a two-step estimation procedure for $k_t$, taking $a_x$ and $b_x$ estimates from the first step as given. We thereby find an estimate of $k_t$ such that, for each year, given the actual population age distribution, the implied number of deaths will equal the actual number of deaths.

On top of estimating the model, we are interested in using it to forecast mortality. Forecasting requires the selection of the dynamics of the evolution in time of the adjusted $k_t$. It is usually modeled using ARIMA time series methods, and, most commonly, as a random walk with drift, i.e. an ARIMA(0,1,0). We perform the required tests and select this hypothesis to forecast our mortality rates as well, estimating appropriately the parameters of the process. Table 3 collects the whole set of estimated parameters.

We then simulate 10,000 paths of $k_t$ for the years 1985-2011 to obtain a simulated distribution of forecasted mortality rates. Simulated $k_t$s are combined with the vectors $a_x$ and $b_x$, estimated according to the procedure described above to produce forecasts of age-specific mortality rates for the years 1985-2011.

The fan-chart in Figure 2 illustrates the historical log mortality rates for the period 1935-1984 (HMD database), and our forecasted distribution of log mortality rates for the period 1985-2011 for a 65 years-old individual. As we can easily see from the figure, the trend of log mortality rates for future years 1985-2011 is decreasing, implying that the observed increases in life expectancy are captured by the model. However, the figure also shows the remarkable uncertainty in the evolution of mortality rates in time, especially when the objective is to forecast their value in a relatively distant future.

Having obtained a model-based forecast of the mortality rates of the Italian population using 1984 data, we can now produce the estimated forecasts of the pension expenditure of the retirees in our sample. These forecasts represent the future scenarios of expenditures that, given the pension payments granted to pensioners, the state may have expected to face according to 1984 actuarial mortality projections.[10]

As a base-line scenario, we consider the age-specific averages of our 10,000 simu-

[9]Even if HMD database provides Italian mortality data since 1900, we chose to discard the period 1900-1935 because it is too far away in time.

[10]Actuarial projections used for state pension calculations have usually been updated in Italy with a frequency of 8 to 10 years, thus justifying to some extent our exercise.

**Table 3:** Estimated parameters of the LC Model

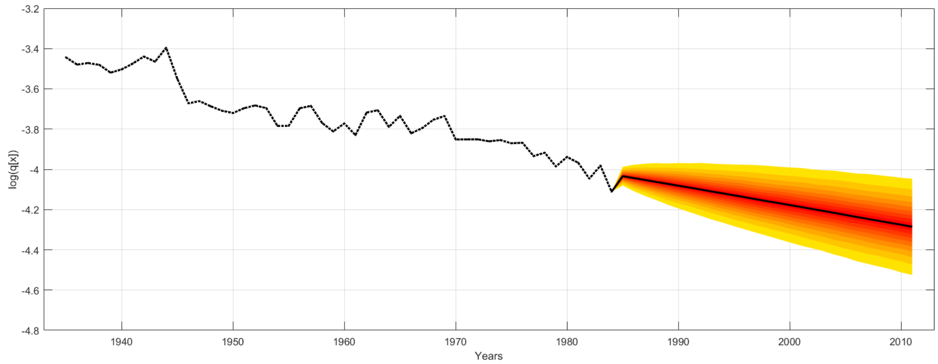
| Parameters $a_x$ and $b_x$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Age | $a_x$ | $b_x$ | Age | $a_x$ | $b_x$ | Age | $a_x$ | $b_x$ |
| 0 | -3,103 | 0,016 | 37 | -6,070 | 0,013 | 74 | -2,814 | 0,004 |
| 1 | -5,339 | 0,031 | 38 | -6,003 | 0,012 | 75 | -2,707 | 0,004 |
| 2 | -6,159 | 0,026 | 39 | -5,942 | 0,011 | 76 | -2,598 | 0,004 |
| 3 | -6,583 | 0,022 | 40 | -5,853 | 0,011 | 77 | -2,496 | 0,004 |
| 4 | -6,841 | 0,020 | 41 | -5,791 | 0,010 | 78 | -2,386 | 0,004 |
| 5 | -7,033 | 0,019 | 42 | -5,715 | 0,010 | 79 | -2,279 | 0,005 |
| 6 | -7,138 | 0,018 | 43 | -5,641 | 0,009 | 80 | -2,180 | 0,004 |
| 7 | -7,257 | 0,018 | 44 | -5,562 | 0,009 | 81 | -2,086 | 0,004 |
| 8 | -7,349 | 0,017 | 45 | -5,470 | 0,008 | 82 | -1,980 | 0,004 |
| 9 | -7,406 | 0,017 | 46 | -5,389 | 0,008 | 83 | -1,879 | 0,004 |
| 10 | -7,443 | 0,018 | 47 | -5,310 | 0,007 | 84 | -1,779 | 0,004 |
| 11 | -7,433 | 0,017 | 48 | -5,221 | 0,006 | 85 | -1,693 | 0,004 |
| 12 | -7,394 | 0,017 | 49 | -5,141 | 0,006 | 86 | -1,600 | 0,004 |
| 13 | -7,324 | 0,016 | 50 | -5,044 | 0,006 | 87 | -1,513 | 0,004 |
| 14 | -7,172 | 0,015 | 51 | -4,968 | 0,005 | 88 | -1,419 | 0,004 |
| 15 | -7,016 | 0,015 | 52 | -4,886 | 0,005 | 89 | -1,322 | 0,004 |
| 16 | -6,868 | 0,014 | 53 | -4,806 | 0,005 | 90 | -1,256 | 0,003 |
| 17 | -6,767 | 0,014 | 54 | -4,716 | 0,004 | 91 | -1,183 | 0,003 |
| 18 | -6,606 | 0,017 | 55 | -4,637 | 0,004 | 92 | -1,100 | 0,003 |
| 19 | -6,535 | 0,018 | 56 | -4,548 | 0,004 | 93 | -1,024 | 0,003 |
| 20 | -6,508 | 0,020 | 57 | -4,465 | 0,004 | 94 | -0,953 | 0,003 |
| 21 | -6,478 | 0,020 | 58 | -4,379 | 0,004 | 95 | -0,898 | 0,003 |
| 22 | -6,466 | 0,020 | 59 | -4,290 | 0,004 | 96 | -0,825 | 0,003 |
| 23 | -6,447 | 0,020 | 60 | -4,194 | 0,004 | 97 | -0,774 | 0,002 |
| 24 | -6,447 | 0,019 | 61 | -4,114 | 0,003 | 98 | -0,705 | 0,003 |
| 25 | -6,445 | 0,019 | 62 | -4,013 | 0,004 | 99 | -0,572 | 0,002 |
| 26 | -6,451 | 0,019 | 63 | -3,925 | 0,004 | 100 | -0,598 | 0,002 |
| 27 | -6,453 | 0,018 | 64 | -3,832 | 0,004 | 101 | -0,557 | 0,002 |
| 28 | -6,445 | 0,017 | 65 | -3,736 | 0,004 | 102 | -0,539 | 0,003 |
| 29 | -6,428 | 0,017 | 66 | -3,643 | 0,004 | 103 | -0,429 | 0,004 |
| 30 | -6,387 | 0,017 | 67 | -3,545 | 0,004 | 104 | -0,466 | 0,004 |
| 31 | -6,370 | 0,016 | 68 | -3,443 | 0,004 | 105 | -0,223 | 0,009 |
| 32 | -6,331 | 0,015 | 69 | -3,341 | 0,004 | 106 | -0,207 | 0,006 |
| 33 | -6,287 | 0,015 | 70 | -3,233 | 0,004 | 107 | 0,022 | 0,006 |
| 34 | -6,241 | 0,014 | 71 | -3,138 | 0,004 | 108 | 0,096 | 0,000 |
| 35 | -6,190 | 0,014 | 72 | -3,026 | 0,004 | 109 | 0,185 | -0,003 |
| 36 | -6,124 | 0,013 | 73 | -2,921 | 0,004 | 110 | 0,052 | -0,001 |

| Parameter $k_t$ ARIMA (0,1,0) Model | |
|---|---|
| Drift | Variance |
| -2,54492 | 54,8979 |
| (1.03341) | (13.0598) |

Source: Authors' calculation

lations. We use these average log-mortality rates to calculate the age-specific survival rates by cohort of birth and we apply them to the pension expenditure by cohort, as explained in the previous subsection. As a result, we obtain the total pension expenditure by year of birth, discounted by the survival rates so obtained, from the retirement

**Figure 2:** LC forecasted log mortality rates at age 65: years 1985-2011




Source: Authors' calculation

date up to 2011.

As before, aggregating by cohorts and years, we obtain the total expected payment for the whole sample of pensioners up to 2011, which is 6,208,473,761 Euros. Comparing this figure with our estimate of the realized pension expenditure, we find that this estimate is 2.30% lower. This provides us with a measure of the effects of longevity risk on our sample: the best estimate of the expenditure, that is the amount that, given the profile of pension payments, the state expected to pay to retirees, was indeed about 150 million Euros lower than the actual payment. This underestimation is the result of an unpredicted rise in the average life expectancy of pensioners, that even a rather sophisticated actuarial model such as the LC one could not capture.

Now, instead of taking the average of the 10,000 possible forecasted age-specific log mortality rates for the period 1985-2011, we take different percentiles of such log mortality rates. Taking a low percentile of the simulated distribution implies assuming a worst-case scenario for the annuity provider, considering that the retirees will experience mortality rates lower than the expectations. Table 4 presents the forecasted expenditures, total and by year of retirement, obtained selecting different percentiles of our simulated age-specific mortality rates, and compares them with the actual estimate presented in Section 2.1. Underestimation of the expenditure is obtained even when selecting a very low percentile. At the $5^{th}$ percentile, representing a 1 over 20 years possible scenario, the forecasted overall expenditure is 1.04% lower than our estimate of the actual one. It is interesting to notice that this underestimation is obtained due to a too low variability of the mortality rates in the long run. Indeed, the estimates that are more distant, such as the expenditure of the 2004 retirees, are too low with respect to their actual counterparts. This is due to an excessively small variability in the forecasted mortality rates produced by the model. We now turn to analyze possible improvements of the forecasts, to see whether and how they may limit the longevity risk issue we highlighted.

**Table 4:** Forecasted pension expenditure (in Euro) with the LC model: base year 1984

| Year of Retirement | LC Average | LC $5^{th}$ %ile | LC $10^{th}$ %ile | LC $30^{th}$ %ile | Realized |
|---|---|---|---|---|---|
| 1985 | 199,003,999 | 203,498,775 | 202,569,630 | 200,508,528 | 200,969,315 |
| 1986 | 231,506,287 | 236,613,552 | 235,557,111 | 233,218,222 | 234,512,955 |
| 1987 | 262,951,867 | 268,333,305 | 267,220,433 | 264,759,766 | 267,027,489 |
| 1988 | 289,022,079 | 294,798,664 | 293,603,204 | 290,965,105 | 294,324,699 |
| 1989 | 297,727,644 | 303,344,935 | 302,181,972 | 299,619,623 | 303,963,558 |
| 1990 | 320,982,986 | 326,687,311 | 325,506,133 | 322,906,110 | 328,435,410 |
| 1991 | 343,192,847 | 348,868,478 | 347,693,372 | 345,108,364 | 351,769,440 |
| 1992 | 510,067,308 | 517,470,754 | 515,939,554 | 512,569,006 | 522,936,015 |
| 1993 | 260,322,025 | 264,217,705 | 263,411,596 | 261,637,648 | 267,478,388 |
| 1994 | 494,472,861 | 500,804,655 | 499,497,137 | 496,611,973 | 508,229,840 |
| 1995 | 368,551,735 | 372,732,006 | 371,869,990 | 369,962,938 | 378,347,963 |
| 1996 | 454,250,861 | 458,986,580 | 458,010,043 | 455,847,276 | 466,544,569 |
| 1997 | 407,383,629 | 411,324,129 | 410,509,996 | 408,709,932 | 418,508,243 |
| 1998 | 305,938,131 | 308,537,065 | 307,999,116 | 306,811,993 | 313,819,978 |
| 1999 | 323,165,190 | 325,710,842 | 325,181,396 | 324,019,458 | 331,606,150 |
| 2000 | 246,673,676 | 248,419,700 | 248,054,577 | 247,258,290 | 252,950,271 |
| 2001 | 257,989,299 | 259,620,595 | 259,277,519 | 258,534,137 | 264,320,748 |
| 2002 | 222,486,746 | 223,697,787 | 223,442,054 | 222,890,445 | 227,568,902 |
| 2003 | 207,130,147 | 208,124,842 | 207,913,817 | 207,461,007 | 211,640,947 |
| 2004 | 205,654,444 | 206,466,042 | 206,293,604 | 205,924,037 | 209,655,464 |
| Total | 6,208,473,761 | 6,288,257,723 | 6,271,732,255 | 6,235,323,860 | 6,354,610,346 |
| Underestimation | 2.30% | 1.04% | 1.30% | 1.88% | - |

Source: Authors' calculation

## 3. Robustness

In this Section, we consider two robustness checks to our results, when improving the forecasts in two directions. First, instead of using a fixed 1984-based forecast of mortality rates for the whole period 1985-2011, we update the estimate of the mortality rate LC model. Second, we try to forecast the mortality rates of the ages 60-90 with the Cairns et al. (2006), that is more appropriate to describe the features of those specific ages.

### *3.1. Forecasted pension expenditure: yearly updates of LC estimates of mortality rates*

As a first robustness check for our results, we update our estimates of the LC model every year starting from 1985 to 2004 (retirement window of sample retirees) and use these estimates to produce updated forecasts each year. More specifically, we use the 1984 forecast to compute the State pension expenditure borne in the period 1985-2011 for those retired in 1985. Then, we use the 1985 forecast to calculate the State expenditure of retirement benefits paid in the period 1986-2011 to those retired in 1986, and so on. This process is consistent with the idea that, in principle, a public pension provider could fix the initial pension amount for his pensioners using an actuarial fairness principle and basing the estimates of their expected residual lifetimes on information available up to the moment of retirement. Indeed, these forecasts should obviously be more precise than the ones obtained in the previous section, when the mortality model was estimated only once and for all at the beginning of the observation period. This idea is in line with some current automatic balancing mechanisms, that automatically adjust retirement age/conversion factors to life expectancy improvements at specified intervals (see Godínez-Olivares et al. (2016)). Table 5 collects the results of this exercise. As expected, the yearly update of the estimates improves the results. However, the average total forecasted pension expenditure for the period 1985-2011 obtained by applying a yearly estimation of mortality rates amounts to 6,315,525,752 Euros, i.e. the 0.61% lower than the actual pension expenditure. This figure is indeed closer to our estimate of the realized expenditure, but still evidences that even the yearly update of the forecasts does not immunize entirely against the risk of unexpected longevity rises. Indeed, it would be necessary to consider a "prudent" percentile of the distribution of mortality rates ranging between 10 and 30 to match the realized expenditure. This is important to keep in mind, as recent pension reforms attached the revision of pension amounts and retirement age to longevity improvements. This obviously reduces the risk of running into higher-than-predicted costs, but does not entirely avoid it.

#### *3.1.1. Forecasted pension expenditure: mortality rates from the Cairns et al. (2006) model*

In this section, we improve our estimates by describing the evolution of mortality at ages 60-90 using the CBD model. As already mentioned, differently from the LC model, the CBD model was developed with the aim of providing more accurate mortality projections for older ages, who constitute indeed the bulk of the pension expenditure of each pension system or pension fund. Indeed, according to Biffi and Clemente (2014), the CBD model is the best approach to model mortality at high ages for Italy.

According to the CBD model, the probability of death, $q_{x,t}$ is described by the following expression:

$$logit(q_{x,t}) = k_t^{[1]} + k_t^{[2]}(x - \bar{x}) + e_{x,t}. \tag{2}$$

The logit of the age-specific probability of death is modeled as a linear function of age $x$, where $\bar{x}$ represents the mean age in the sample range. The intercept $k_t^{[1]}$

**Table 5:** Forecasted pension expenditure (in Euro) with the LC model: yearly updates of estimates

| Year of Retirement | $LC_{yu}$ Average | $LC_{yu}$ $5^{th}$ %ile | $LC_{yu}$ $10^{th}$ %ile | $LC_{yu}$ $30^{th}$ %ile | Realized |
|---|---|---|---|---|---|
| 1985 | 198,951,584 | 203,524,721 | 202,581,905 | 200,441,475 | 200,969,315 |
| 1986 | 231,413,279 | 236,573,692 | 235,484,186 | 233,153,730 | 234,512,955 |
| 1987 | 264,002,148 | 269,294,079 | 268,209,364 | 265,711,703 | 267,027,489 |
| 1988 | 292,029,682 | 297,541,217 | 296,361,615 | 293,827,020 | 294,324,699 |
| 1989 | 301,151,466 | 306,395,774 | 305,300,843 | 302,865,779 | 303,963,558 |
| 1990 | 326,257,527 | 331,475,194 | 330,379,981 | 327,967,859 | 328,435,410 |
| 1991 | 348,962,503 | 354,010,003 | 352,941,029 | 350,675,867 | 351,769,440 |
| 1992 | 518,593,723 | 525,057,801 | 523,575,155 | 520,790,381 | 522,936,015 |
| 1993 | 266,228,471 | 269,435,821 | 268,774,313 | 267,288,609 | 267,478,388 |
| 1994 | 505,052,213 | 510,120,673 | 508,989,100 | 506,743,869 | 508,229,840 |
| 1995 | 376,966,482 | 379,350,347 | 378,658,352 | 377,135,156 | 378,347,963 |
| 1996 | 463,818,094 | 467,306,918 | 466,510,026 | 464,910,444 | 466,544,569 |
| 1997 | 416,521,617 | 419,479,038 | 418,807,802 | 417,529,296 | 418,508,243 |
| 1998 | 312,490,960 | 314,382,397 | 313,973,988 | 313,133,220 | 313,819,978 |
| 1999 | 329,909,721 | 331,811,893 | 331,410,513 | 330,530,770 | 331,606,150 |
| 2000 | 251,997,377 | 253,312,423 | 253,020,877 | 252,442,193 | 252,950,271 |
| 2001 | 263,818,196 | 264,971,389 | 264,734,704 | 264,216,324 | 264,320,748 |
| 2002 | 227,382,182 | 228,208,336 | 228,043,633 | 227,656,621 | 227,568,902 |
| 2003 | 211,556,869 | 212,239,414 | 212,099,413 | 211,776,576 | 211,640,947 |
| 2004 | 209,321,648 | 209,886,047 | 209,771,608 | 209,504,059 | 209,655,464 |
| Total | 6,315,525,752 | 6,384,377,187 | 6,369,628,416 | 6,338,278,105 | 6,354,610,346 |
| Underestimation | 0.61% | −0.46% | −0.23% | 0.25% | - |

Source: Authors' calculation

and the slope $k_t^{[2]}$ are stochastic processes. We select them to be correlated random walks, as in the original formulation of the model. The former affects every age in the same way, while the impact of $k_t^{[2]}$ varies according to age, being higher for ages distant from the average age in the sample, i.e. 75. The error term $e_{x,t}$ reflects the historical (age-specific) patterns not captured by the model.

We apply this model to forecast the mortality rates of the ages 60-90 in our sample, while we maintain our original LC estimates and forecasts for the other ages. We thus estimate the model using the Italian mortality data from 1935 to 1984, estimating $k_t^{[1]}$ and $k_t^{[2]}$, whose values are collected in Table 6.

The trend of $k_t^{[1]}$ is reducing overtime, implying that the mortality rates have been decreasing at all ages. Conversely, the values of $k_t^{[2]}$ are increasing overtime, meaning that the mortality improvements have been greater at medium ages (around 60) and at higher ages rather than at central ones (around 75) in recent years.

Given the estimated parameters, we simulate $k_t^{[1]}$ and $k_t^{[2]}$ and combine them according to Equation 2 to produce forecasts of mortality rates in the age range 60-90 for the period 1985-2011. The fan-chart in Figure 3 illustrates the historical log mortality rates for the period 1935-1984 (HMD database), and the 10,000 forecasted log mortality rates

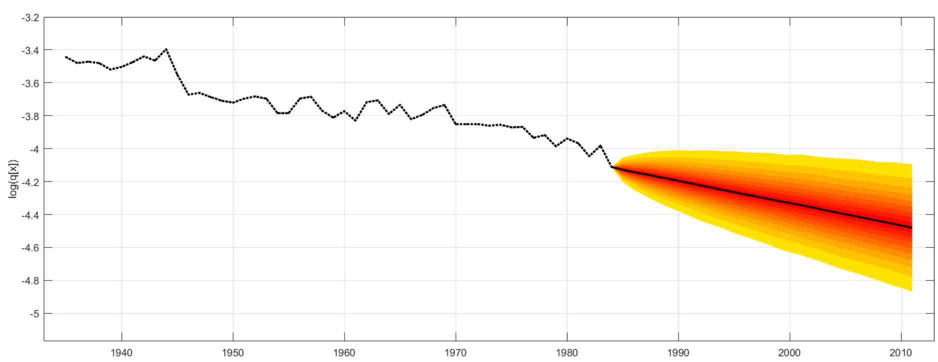
**Table 6:** Estimated parameters $k_t^{[1]}$ and $k_t^{[2]}$ of the CBD model

| Year | $k_t^{[1]}$ | $k_t^{[2]}$ | Year | $k_t^{[1]}$ | $k_t^{[2]}$ |
|---|---|---|---|---|---|
| 1936 | -2,343531916 | 0,109697942 | 1961 | -2,712524269 | 0,1125323 |
| 1937 | -2,337586844 | 0,108065883 | 1962 | -2,628668625 | 0,114938986 |
| 1938 | -2,328644817 | 0,113056448 | 1963 | -2,631985253 | 0,113833747 |
| 1939 | -2,342518962 | 0,115210143 | 1964 | -2,717645656 | 0,10988918 |
| 1940 | -2,283113054 | 0,117805985 | 1965 | -2,660982155 | 0,112455753 |
| 1941 | -2,279204176 | 0,119428198 | 1966 | -2,735162324 | 0,110999325 |
| 1942 | -2,248999222 | 0,119224772 | 1967 | -2,719640192 | 0,111166669 |
| 1943 | -2,265095499 | 0,116478356 | 1968 | -2,681111379 | 0,113701263 |
| 1944 | -2,242983598 | 0,11920428 | 1969 | -2,691542995 | 0,1036612 |
| 1945 | -2,377431071 | 0,116593622 | 1970 | -2,750169083 | 0,108895078 |
| 1946 | -2,500578739 | 0,115383117 | 1971 | -2,767043494 | 0,110139067 |
| 1947 | -2,505131407 | 0,112378237 | 1972 | -2,789009666 | 0,108621377 |
| 1948 | -2,548121417 | 0,110889088 | 1973 | -2,748810592 | 0,113527407 |
| 1949 | -2,537011453 | 0,112915383 | 1974 | -2,797958662 | 0,111896011 |
| 1950 | -2,603186258 | 0,111832384 | 1975 | -2,759335907 | 0,111635557 |
| 1951 | -2,508076906 | 0,115300696 | 1976 | -2,784950585 | 0,112267495 |
| 1952 | -2,518831522 | 0,116950192 | 1977 | -2,813412999 | 0,11335119 |
| 1953 | -2,514952372 | 0,11526673 | 1978 | -2,843992739 | 0,111612077 |
| 1954 | -2,635455672 | 0,113325938 | 1979 | -2,871439288 | 0,111991897 |
| 1955 | -2,642625361 | 0,112552946 | 1980 | -2,857448758 | 0,112071886 |
| 1956 | -2,496106561 | 0,118907274 | 1981 | -2,882303731 | 0,112304065 |
| 1957 | -2,580358018 | 0,111635482 | 1982 | -2,928126908 | 0,110904174 |
| 1958 | -2,666976861 | 0,111250593 | 1983 | -2,894531256 | 0,114825542 |
| 1959 | -2,708532356 | 0,110807465 | 1984 | -2,97734246 | 0,112089278 |
| 1960 | -2,652875548 | 0,114798641 | | | |

Source: Authors' calculation

for the period 1985-2011 of a 65 years-old individual. One can easily see, by comparing Figure 3 and Figure 2, that the CBD model produces forecasts that are more variable than those obtained with the LC model.

**Figure 3:** CBD forecasted log mortality rates at age 65: years 1985-2011




Source: Authors' calculation

Following the same procedure described in the previous sections, we compute the pension expenditures in an average scenario and different percentile-forecasts of mortality rates. Table 7 collects the total forecasted expenditure and the forecasts for each cohort of retirees, under different mortality scenarios. In the average scenario, the forecasted expenditure for the whole sample of individuals is 6,257,819,903 Euros, 1.52% lower than the actual pension expenditure. The larger variability in the mortality dynamics captured by the CBD model relative to the LC one leads to a better result on the tail scenarios: when the $5^{th}$ percentile of each age-specific mortality rate is considered, the forecasted expenditure exceeds the realized one by 0.25%.

**Table 7:** Forecasted pension expenditure (in Euro) with the CBD model: base year 1984

| Year of Retirement | CBD Average | CBD $5^{th}$ %ile | CBD $10^{th}$ %ile | CBD $30^{th}$ %ile | Realized |
|---|---|---|---|---|---|
| 1985 | 201,105,097 | 207,877,143 | 206,499,849 | 203,400,992 | 200,969,315 |
| 1986 | 233,962,706 | 241,630,518 | 240,071,462 | 236,564,002 | 234,512,955 |
| 1987 | 265,636,855 | 273,590,700 | 271,975,438 | 268,340,381 | 267,027,489 |
| 1988 | 292,009,424 | 300,514,859 | 298,789,413 | 294,902,526 | 294,324,699 |
| 1989 | 300,732,432 | 308,928,064 | 307,265,183 | 303,522,516 | 303,963,558 |
| 1990 | 324,147,870 | 332,387,642 | 330,715,271 | 326,955,491 | 328,435,410 |
| 1991 | 346,484,665 | 354,595,258 | 352,949,317 | 349,250,787 | 351,769,440 |
| 1992 | 514,543,141 | 524,874,894 | 522,777,993 | 518,070,314 | 522,936,015 |
| 1993 | 262,709,730 | 268,272,753 | 267,144,008 | 264,605,817 | 267,478,388 |
| 1994 | 498,748,113 | 507,504,533 | 505,725,706 | 501,734,426 | 508,229,840 |
| 1995 | 371,387,905 | 377,092,234 | 375,931,738 | 373,331,190 | 378,347,963 |
| 1996 | 457,652,533 | 464,056,633 | 462,752,553 | 459,831,409 | 466,544,569 |
| 1997 | 410,312,806 | 415,662,556 | 414,570,586 | 412,128,305 | 418,508,243 |
| 1998 | 307,873,733 | 311,352,741 | 310,640,576 | 309,052,233 | 313,819,978 |
| 1999 | 325,152,307 | 328,577,106 | 327,874,216 | 326,308,289 | 331,606,150 |
| 2000 | 248,101,720 | 250,443,955 | 249,961,761 | 248,889,815 | 252,950,271 |
| 2001 | 259,367,580 | 261,546,291 | 261,096,104 | 260,098,145 | 264,320,748 |
| 2002 | 223,505,240 | 225,105,233 | 224,773,223 | 224,039,984 | 227,568,902 |
| 2003 | 208,025,096 | 209,342,129 | 209,067,703 | 208,463,912 | 211,640,947 |
| 2004 | 206,360,949 | 207,412,550 | 207,191,488 | 206,710,370 | 209,655,464 |
| Totals | 6,257,819,903 | 6,370,767,792 | 6,347,773,584 | 6,296,200,901 | 6,354,610,346 |
| Underestimation | 1.52% | -0.25% | 0.11% | 0.92% | - |

Source: Authors' calculation

## 4. Comments and Conclusions

In this paper we provided evidence about the relevance of longevity risk for public pension issuers. Based on Italian data, we quantified a severe underestimation of mortality rates over a 30 year span, when forecasts are based on central estimates provided by sound actuarial stochastic mortality models. Our results show that, in our sample, not only actual pension payments have exceeded expectations, but that actual expenditure turned out to be a very low percentile of the forecasted distribution.

Our real-world based experiment clarifies the importance of considering the uncertainty in predictions. Even though longevity is perceived as a threat that builds slowly over time, it is now general consensus that it needs to be tackled, to prevent further deterioration of countries financial stability. Measures able to avoid potential large negative effects on private and public sector finances are needed. The debate on how to reduce the risks related to aging and to the underestimation of longevity improvements for public pensions has recently gained attention. IMF (2012), for instance, proposes several potential solutions, aimed at reducing the threat that longevity risk represents for financial viability. First, longevity risk, to which public pension systems are highly exposed, as we showed in our results, can be shared to some extent between the private sector, the public sector, and individuals, through pension reforms. An example of such sharing mechanism is retirement age indexation to life expectancy improvements, that has been adopted recently by some countries, such as Italy. A more complex, but effective, solution may consist in allowing governments, individuals and private pension providers to transfer the longevity risk to capital market participants and private companies (see for instance Blake and Burrows (2001) and Blake et al. (2014)). However, most importantly, a careful investigation about the uncertainty regarding mortality rates forecasts is necessary. A correct, threat-minimizing valuation of pension benefits appears to be crucial for public pension systems, just as well as for private ones, in reducing financial instability issues.

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