The Patent Quality Control Process: Can We Afford An (Rationally) Ignorant Patent Office?

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Abstract

This paper considers patent granting as a two-tiered process, which consists of patent office examination (public enforcement) and court challenges (private enforcement). It argues that, when the patent-holder has private information about the patent validity, (i) a weak patent is more likely to be settled and thus escape court challenges than a strong patent; and (ii) when the economy suffers from the low patent quality problem, a tighter examination by the patent office may strengthen private scrutiny over a weak patent. Both work against Lemley (2001)'s hypothesis of a "rationally ignorant" patent office.

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1 Introduction

Patent quality, defined as the extent to which issued patents conform to patent law requirements,[1] has been one of the dominant concerns about the "broken" United States patent system over the past 10-15 years (Jaffe and Lerner, 2004). The flood of weak patents, i.e., those patents don't deserve patent protection, is accused of damaging innovation (Jaffe and Lerner, 2004, Bessen and Meurer, 2008) and hampering post-innovation market efficiency (Farrell and Shapiro, 2008). Dissatisfaction and cautions have been raised by industry stake-holders, academic commentators, and government agencies such as the U.S. Federal Trade Commission and the Antitrust Division of the Department of Justice (FTC, 2003). In response, the U.S. Congress has initiated a series of legislation efforts beginning in 2005; the most recent bills proposed to reform the U.S. patent system were introduced in March 2009.[2]

When identifying the source of the problem, there is a consensus that the current "crisis" is largely attributable to lax quality control in the U.S. Patent and Trademark Office (hereafter, USPTO). However, when talking about reform, disproportional attention seems to be shifted away from the patent office. For instance, recent reform bills remain silent on how to improve the performance of the USPTO. This "ignorance" on the part of the patent office might be based on a good reason. As suggested by Lemley (2001)'s influential "rational ignorance" argument, the patent office may optimally set its examination standard at a not-so-high level, even though quite a few patents with questionable quality would be issued.

Lemley (2001)'s "rational ignorance" hypothesis is based on two premises. First, the patent granting decision is in fact structured as a two-tiered process. Besides the inspection by patent office examiners (the public enforcement tier), private parties can also challenge the validity of issued patents in court or at the patent office (the private enforcement tier).[3] And second, private challengers have considerable advantages over

[1]In most jurisdictions, a patent is granted to an invention that is novel, non-obvious (or contains an inventive step), and useful (or has industrial applications). The first two are technological criteria, i.e., the comparison is made between the invention and existing technologies. The usefulness criterion, in practice, is also determined by whether the invention has any applications, but not whether it is profitable in the market.

[2]The major sponsors are Senators Orrin Hatch, Patrick Leahy (S. 515), and Jon Kyl (S. 610) in the Senat; and Representative John Conyers (H.R. 1260) in the House. On April 2, 2009, the Senate Judiciary Committee approved to bring S. 515 to the full Senate. These bills closely follow earlier proposed legislation including the Patent Reform Act of 2007 and the Patent Reform Act of 2005.

[3]The post-grant challenge procedures within the patent office are called patent reexamination in the U.S., and patent opposition in Europe, respectively.

the public agency in the process. They have more knowledge about which patents cover valuable inventions, so the granted monopoly entails serious consequences; they also closely follow technological developments and have more information about where to locate those prior arts crucial to patentability evaluations. Under these two presumptions, Lemley (2001) argues that, instead of carefully scrutinizing every patent application at the patent office, it would be more efficient to lower the examination standard and issue some patents with questionable quality, while letting private parties select which patents to dispute in detail in court. A glance at recent legislation proposals also reveals this emphasis on the private sector to eliminate weak patents.[4]

In this paper, we accept the two premises. Nevertheless, we argue that to embrace this hypothesis for policy guidance, at least two questions have to be addressed: How reliable is private enforcement in improving the patent quality? And how would public enforcement affect private behavior? More specifically, we consider whether and when a private challenger would initiate a validity challenge against the "right target," i.e., weak patents, rather than settle the case; and whether better patent office performance would strengthen or weaken private incentives to litigate and weed out weak patents.

To answer the two questions, we consider a bargaining game between a patent-holder/inventor and a private challenger: Before launching a validity challenge, a patent-holder and a potential challenger engage in pre-trial settlement bargaining, and this bargaining is clouded with asymmetric information. The key to our analysis is the well-known case selection effect in the law and economics literature, namely, the systematic difference between those settled and unsettled disputes. In Section 2, we introduce a simple two-type model where the patent-holder has private information about her patent being a strong or weak type, and the challenger optimally chooses the litigation efforts exercised in court should bargaining break down. A strong patent is assumed to be possessed by a true inventor. By contrast, the owner of a weak patent tries to game the system and patent technologies in the public domain.

In Section 3 we show that bargaining breakdown is more likely to happen and a challenge ensue when the dispute involves a strong patent, for its owner will be "tougher" at the bargaining table. For the same level of litigation efforts, a strong patent is more likely to withstand a challenge than a weak patent. Private force, then,

[4]For instance, the 2009 version of proposed reform follows its previous versions to include a post-grant review procedure and the possibility for third parties to submit documents relevant to the examination of a patent application. The latter is similar to a pre-grant challenge procedure that we shall consider in Appendix B.

may be exerted toward the wrong target, and the true inventor may face a higher litigation risk than the opportunistic player.

Even when the weak patent can be eliminated by private challenges, it doesn't necessarily imply that we can rely on private force to such an extent that the patent office could "delegate" the task to private players while reducing or maintaining low examination standards. In Section 4, we show that public and private enforcement may exhibit a non-monotonic relationship. When patent quality is sufficiently high, a better-functioning patent office may crowd out private incentives to strike down the weak patent. However, when patent quality is sufficiently low, a greater effort at the patent office may *increase* the chance of eliminating the weak patent in court. In this case, strengthening the patent office's performance creates a multiplying effect by enlisting more private force against the weak patent. Together with the case selection pattern, these results cast doubts on the "rational ignorance" hypothesis and caution the danger of maintaining a "weak" patent office. Put differently, we provide a *raison-d'être* for the patent office, and refute the idea of abolishing patent office examination and moving toward a patent registration system.[5]

A caveat is in order: Our analysis shows how the patent office performance should be adjusted in order to increase patent quality, but does not address the issue of the optimal patent quality. That is, we do not perform a full-fledged welfare analysis to pin down the optimal patent quality and the corresponding examination effort exerted by the patent office. Doing so would require the construction of a social welfare function as a function of patent quality, which in turn demands further details such as the static deadweight loss associated with the patent rights, the dynamic incentives the patent could generate, and the cost disadvantage of the patent office *vis-à-vis* the private challenger, *etc.*. Nevertheless, our results directly apply whenever low patent quality is a dominant policy concern. For instance, when the social loss due to weak patents far outweighs the additional expenditure or the cost of other policy measures to enhance the performance of the patent office, then raising patent quality becomes the major policy objective.

We conclude the paper in Section 5 with a discussion of future research. All proofs are relegated to Appendix A. In Appendix B we consider the effect of a pre-grant challenge system. (Appendix C, not for publication, provides robustness checks of our main results in alternative settings.)

[5]See Merges (1999).

☐ **Related literature:** Recent concerns about patent quality have spurred reform proposals from different sources.[6] These proposals cover a wide range of issues, but often lack formal analysis. One reason, perhaps, is that relative to the optimal design in terms of patent length, scope, and other policy instruments, only recently have scholars started the theoretical efforts toward patent examination, or more generally the implementation of the patent system. Kesan and Gallo (2006) describes how weak patents can be settled in a symmetric information environment with legal expenses. Other papers, such as Langinier and Marcoul (2008), Caillaud and Duchêne (2009), Prady (2008), and Schuett (2009) elaborate on the strategic interaction between patent applicants and the patent office. Unlike these papers, we emphasize the "second eye," that is, the role of private enforcement, and consider the cooperation between public and private sectors to improve patent quality. In this aspect, Koenen (2009) adopts a similar framework as our paper, but explores different private strategies. While we allow for a private challenger to adjust his litigation efforts in court, Koenen (2009) excludes this decision but instead lets the challenger choose whether to invalidate the patent or simply infringe and enter the market.

There is extensive discussion in the law and economics literature about the division of labor between public and private enforcement, both in a general framework (Shavell, 1993) and in specific fields such as antitrust law (Segal and Whinston, 2006, Bourjade *et al.*, 2009). In particular, Briggs *et al.* (1996) and Bourjade *et al.* (2009) also incorporate case selection into antitrust enforcement.[7] In fact, in a two-stage enforcement structure, Briggs *et al.* (1996) analyzes case selection in *both* public and private enforcement. That is, they allow the public antitrust agency to settle the case, a common practice in the antitrust context, and stress how the occurrence or disappearance of a subsequent private (treble damage) suit would affect settlement behavior at the public enforcement stage.[8] By contrast, we do not permit the patent office to "settle" with a patent applicant, and focus more on how public enforcement will affect private enforcement. Bourjade *et al.* (2009), similar to our analysis, raises the case se-

[6]E.g., FTC (2003), NAS (2004). Also see the special issue of *Berkeley Technology Law Journal*, 2004, 19 (3).

[7]McAfee *et al.* (2008) points out another strategic element and argues that a private firm may abuse the court avenue and initiate an antitrust lawsuit against its competitor even when the latter doesn't commit any anti-competition action.

[8]In Briggs *et al.* (1996), public enforcement may fully crowd out private enforcement. Due to a fixed litigation cost the private plaintiff may lose the credibility to sue after the government action, despite the defendant's private information. In our model, private litigation disappears only when there is symmetric information between the two private parties and thus no bargaining friction.

lection pattern as a limit of private enforcement, but then moves on to discuss whether and how to facilitate private litigation by adjusting damage rewards or litigation cost; there is no public enforcement in their framework.

More importantly, both Briggs *et al.* (1996) and Bourjade *et al.* (2009) adopt an assumption of what we call "exogenous litigation outcome." That is, once settlement bargaining breaks down and the lawsuit reaches the court (possibly after paying a fixed litigation expenditure), the disputing parties have no further influence over the litigation outcome. In this paper, we relax this assumption and let the private challenger (who tries to invalidate the patent in court) make an optimal litigation expenditure or effort decision, which will in turn affect the chances of striking down the patent. This "endogenous litigation outcome" scenario differentiates our model from the majority of previous case selection studies based on asymmetric information (Spier (2005) and Daughtey and Reinganum (2008) are two excellent review of this literature).[9] Although studies such as Katz (1988) have examined litigants' optimal legal expenditure decisions in court, these models often start directly with litigation and ignore settlement; and most case selection literature have overlooked the expenditure decision in court. Two exceptions are Franzoni (1999) and Friedman and Wickelgren (2008).[10] These two papers focus on different issues and adopt the screening paradigm of Bebchuk (1984), where the uninformed party makes the settlement party.[11] Our signaling model

[9]In the literature, case selection has been extensively studied under two prominent approaches, that of "divergent expectations" and "asymmetric information." A seminal paper using divergent expectations is Priest and Klein (1984). Theoretical research on the asymmetric information paradigm has been well developed in several directions, including one-sided asymmetric information where either the uninformed party makes the offer (screening, Bebchuk (1984), P'ng (1983)), or the informed party makes the offer (signaling, Reinganum and Wilde (1986)); two-sided asymmetric information; and the dynamic multiple-offer bargaining situation, *etc.*. Meurer (1989) applies the asymmetric information paradigm to patent litigation. Empirically, each paradigm can find its support. For instance, Waldfogel (1998) favors the divergent expectations story, while Froeb (1993) supports the asymmetric information approach. We adopt the asymmetric information paradigm on the ground that the low patent quality problem can be alleviated through discouraging applications on technologies already in the public domain, a complaint widely shared, among others, in the software industry. A natural modeling strategy is to consider a situation where the patent applicant, but not other parties, is aware of this gaming behavior, and public policy should address this opportunism. In Section 5 we offer some thoughts about using other approaches to model settlement bargaining.

[10]Gong and McAfee (2000) also considers a two-stage game with bargaining under two-sided private information proceeding litigation, and allow litigants choose their legal expenditure in court. However, the authors exclude Bayesian learning after bargaining breaks down, and assume that if no settlement agreement is reached, then the two parties learn the true probability of litigation outcomes and so there is no more private information in the litigation subgame.

[11]Franzoni (1999) illustrates how settlement may hurt the deterrence objective of legal enforcement. Friedman and Wickelgren (2008) considers the trade-off between maintaining deterrence and avoiding false conviction (the chilling effect).

à la Reinganum and Wilde (1986), where the informed party makes settlement offer, turns out to be a non-trivial difference. At the private bargaining stage, it allows us to obtain a "no-settlement" subgame equilibrium under which private parties cannot reach any settlement agreement and the case is certain to be litigated. This equilibrium survives the standard $D1$ refinement requirement and is not obtained by Franzoni (1999) or Friedman and Wickelgren (2008), or any case selection papers bases on asymmetric information that we mentioned above.[12]

2 Model

Consider an inventor (she) seeking patent protection for her invention. Her chance of receiving a patent depends on the examination effort exerted by the patent office. After the patent grant, she tries to enforce her patent rights but encounters a potential challenger (he), who may be able to invalidate the patent in court. Before litigation, however, the two private parties negotiate to settle the case.

We proceed in two steps: We first characterize the outcome of private bargaining, i.e., the case selection pattern, then conduct comparative static analysis to study how the patent office performance affects this pattern. For the second step, we derive insights about how the patent office performance would affect some policy objectives in the presence of the private parties' strategic behavior.

We consider two policy objectives: the overall patent quality that comes out of the two-stage (patent examination and litigation) process; and the total cost incurred by the patent office and the challenger. Given current concerns about the patent quality, we give priority to the first objective. That is, we consider whether a policy adjustment could reduce the overall examination cost without jeopardizing the patent quality. A general welfare account will necessarily incorporate the two concerns. In addition, it also takes into account the impact of patent quality on both the dynamic innovation incentives and the static inefficiency associated with patent rights.[13] Our focus on patent quality then could be justified by the argument that, over the relevant range, the concerns about innovation incentives and static efficiency dominate the examination

[12]Both assumptions of endogenous litigation outcome and signaling are critical to this result. In Example 2, we show that no settlement equilibrium disappears if the challenger's litigation effort is fixed. In Appendix C, we show that it disappears if the uninformed party makes the offer and the informed party plays mixed strategy, as in Franzoni (1999) and Friedman and Wickelgren (2008).

[13]Remark 3 in Section 4 addresses the impact of patent examination on the true inventor's returns from using the patent system and so her R&D incentives.

cost and mandate a higher patent quality.[14]

Suppose that under perfect examination, the inventor's application will be rejected with a probability θ. For instance, the patent examination body (say, the patent office) has full access to all relevant information, and with probability θ a piece of patent-defeating prior art exists which proves that the inventor's invention doesn't satisfy one or several of the patentability requirements. This probability is referred to as the "invalidity" of the patent (when issued). For simplicity, consider a two-type case $\theta \in \{\underline{\theta}, \bar{\theta}\}$, with $0 < \underline{\theta} < \bar{\theta} \leq 1$. (The case of $\underline{\theta} = 0$ will be treated separately, and our main results extend to this special case.)[15] A "true" inventor, or the "good" type, has low invalidity $\underline{\theta}$, or, equivalently, high validity: She spends considerable resources in R&D activities and brings about technological breakthroughs. By contrast, an inventor with high invalidity $\bar{\theta}$ is called the "bad" or "opportunistic" type: She exploits the public domain and tries to patent an "old" technology. We also refer to a patent with high validity $\underline{\theta}$ (low validity $\bar{\theta}$) as a "strong" patent ("weak" patent, respectively). Assume that θ is the inventor's private information, and the challenger holds the initial belief that $Pr(\underline{\theta}) = \alpha$. Implicit in this assumption is an adverse selection setting on the inventor's side where the R&D stage has finished and so what will happen at the patent examination stage does not affect the composition of the two types of inventor. Define $\theta^0 \equiv \alpha\underline{\theta} + (1-\alpha)\bar{\theta}$ as the *ex ante* average invalidity.

We model patent examination as a "search and destroy" process: The patent office and the private challenger can exert costly efforts e_P and $e_C \in [0, 1]$, respectively, to search for the prior art, and the patent protection is denied if and only if the defeating prior art is found. The efforts e_P and e_C are interpreted as the probability to discover the prior art (given existence), and will be called public and private enforcement efforts, respectively. Assume that, conditional on the existence of prior art, the patent office's and the challenger's search results are independent of each other. Given $\theta \in \{\underline{\theta}, \bar{\theta}\}$, the probability to eliminate the inventor's application by the patent office (the private challenger) is $\theta \cdot e_P$ ($\theta \cdot e_C$, respectively). Note that fixing the effort at a strictly positive

[14]Suppose that the social welfare function is well-behaved in that the marginal benefit is decreasing and marginal cost increasing in patent quality. Then, raising patent quality is more likely to be optimal when the quality is low.

[15]A positive probability to deny the true inventor patent protection, $\underline{\theta} > 0$, may come from a "type II" error in the patent examination process. Patentability standards may be inappropriately interpreted such that, for instance, once an invention is realized, others may perceive it as easier to achieve than it actually was. This "hindersight" bias may render a genuine invention "obvious" or lacking an "inventive step," and so patent protection is denied. Alternatively, the patent authority may grant the monopoly rights to a true inventor only with some probability in order to reduce the deadweight loss (Ayres and Klemperer, 1999).

level, say, $e_P > 0$, the weak patent is more likely to be denied patent protection than the strong patent, $\bar{\theta}e_P > \underline{\theta}e_P$. Because we assume that the true inventor has done some genuine research with novel output, while the opportunistic inventor just tries to copy and patent an existing technology, prior art should be more likely to exist in the latter case and so it is natural to consider an examination process exhibiting this feature.[16]

Let the private challenger's search cost be $c(e_C)$, with $c(0) = c'(0) = 0$, $c(1) = c'(1) = \infty$, and c' as well as $c'' > 0$. On the other hand, we assume that the patent office is less efficient than the private challenger, with a search cost $\gamma c(e_P)$, where $\gamma \geq 1$. The cost disadvantage of the patent office may come from two sources. The patent office examiner may have less knowledge about the current state of technology than the private challenger, so it is more costly for the former to locate relevant prior art. Alternatively, as suggested in Lemley (2001), the challenger may have more information about the economic value of a patent than the patent office, and may be able to target only those with significant values. Without access to this information, the patent office may need to exert a uniform examination effort on a large number of patent applications, of which only a portion has any value. This size effect may cost the patent office more to keep the same level of scrutiny as the private challenger on important patents.

Concerning payoffs, regardless of her type, the inventor gets a monopoly profit $\pi > 0$ when receiving the patent protection in the end of the game, and the challenger gets a benefit $b \in (0, \pi)$ when the patent application is rejected. Otherwise the two receive no return. Assume that the two private players are protected by limited liability.

This payoff structure is consistent with a situation where the inventor has no capacity to commercialize the patented invention and relies on the patent rights to receive licensing payments, and the challenger is one of the downstream firms whose products are within the patent scope. Let b be the challenger's profit from the downstream market, and π the total licensing income the inventor can extract. Without patent protection the inventor gets no licensing revenue, and the challenger keeps the whole b. When the inventor receives patent protection, in the absence of asymmetric information or other bargaining frictions at the licensing bargaining, the inventor can extract

[16]If the reverse is true, i.e., for the same examination effort the strong patent is more likely to be eliminated than the weak patent, then patent examination will only deteriorate patent quality. Furthermore, suppose that the inventor has the option to be the good or bad type, i.e., when a moral hazard element is introduced. Such an examination system would perform poorly as an incentive scheme to encourage innovation, for the inventor would be induced to game the system and play the bad (opportunistic) type.

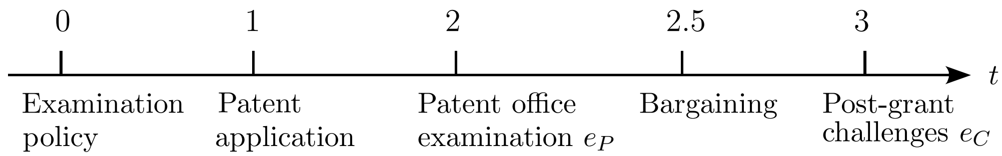

Figure 1: Timing

the whole b from the challenger thanks to full bargaining power. The challenger then gets zero revenue, and as long as there are other licensees, $\pi > b$.[17]

In the two-type case, patent quality can be conveniently defined as the probability that the patent is issued to the true inventor. We are thus concerned with reducing the likelihood of granting patent rights to the opportunistic inventor, whether it's done through private or public efforts.

Figure 1 illustrates the timing of the model:

- At time 0, the patent examination policy is announced;
- at time 1, after observing the policy, the inventor decides whether to file a patent application. The game ends if no application is filed; otherwise,
- at time 2, the inventor's application undergoes examination by the patent office. The game ends if the patent office rejects the application; otherwise the patent is issued, and
- at time 2.5, the inventor and challenger engage in settlement bargaining in order to avoid an invalidation suit and share the monopoly rent π. The game ends if a settlement agreement is reached; otherwise
- at time 3, the challenger exerts a litigation effort e_C and tries to invalidate the inventor's patent in court.

During the pre-trial negotiation, we assign the whole bargaining power to the inventor. She makes a take-it-or-leave-it offer to the challenger.[18]

[17] Suppose instead, that the inventor herself participates at the downstream market and wishes to exclude other competitors, including the challenger. If the inventor receives the patent protection, she will shut down the challenger and all the other firms and get the monopoly profit π, while the challenger will get zero return. If the inventor is denied the patent protection, the inventor will have to to compete with other firms. Suppose that under competition the inventor gets a profit $\tilde{\pi}_I$ and the challenger gets $\tilde{\pi}_C$. Assume that both $\tilde{\pi}_I \geq 0$ and $\tilde{\pi}_C > 0$, and $\pi > \tilde{\pi}_I + \tilde{\pi}_C$. That is, competition will dissipate some, but not all profit, and the challenger is viable under competition. Let $b \equiv \tilde{\pi}_C$. It is easy to see that $\pi > \tilde{\pi}_I + \tilde{\pi}_C \geq \tilde{\pi}_C = b$. For the inventor, we can redefine her litigation payoff and replace expression (2) below with $\tilde{u}_I(\theta, e_C) \equiv (1-\theta e_C)\pi + \theta e_C \tilde{\pi}_I = \tilde{\pi}_I + (\pi - \tilde{\pi}_I)(1-\theta e_C)$, $\theta \in \{\underline{\theta}, \bar{\theta}\}$. Because $\pi - \tilde{\pi}_I > b$, our analysis is robust to this complication.

[18] In Appendix C, we show that our main results are robust to the alternative distribution of bargaining

In the main analysis, we restrict the examination policy to the examination efforts exerted by the patent office, e_P, and assume away any costs of filing the patent application. We consider separately applications fees and the possibility of mounting a private patent challenge at the pre-grant stage in the end of Section 4 and Appendix B, respectively. In the former case, limited liability is dropped out.[19]

3 The Limit of Private Enforcement

In this section we demonstrate that a case involving a weak patent ($\bar{\theta}$) is more likely to be settled than that involving a strong patent ($\underline{\theta}$). This pattern of case selection points out the limit of private enforcement, and is key to subsequent analysis. To determine the bargaining outcome, let us first characterize the threat point, i.e., when no settlement agreement is reached and the two see each other in court.

Suppose that the challenge holds a belief $\tilde{\alpha} \in [0,1]$ that he will encounter the true inventor when trying to invalidate the patent in court. Denote $\tilde{\theta} \equiv \tilde{\alpha}\underline{\theta} + (1-\tilde{\alpha})\bar{\theta} \in [\underline{\theta}, \bar{\theta}]$ as the corresponding expected invalidity. The challenger's optimal litigation effort is determined by

$$e_C^*(\tilde{\theta}) \equiv \arg\max_{e_C} \ \tilde{\theta} e_C b - c(e_C), \tag{1}$$

and the inventor's and challenger's expected litigation payoffs are:

$$u_I(\theta, e_C^*) = (1 - \theta e_C^*)\pi, \quad \theta \in \{\underline{\theta}, \bar{\theta}\} \tag{2}$$

$$u_C(\tilde{\theta}) = \tilde{\theta} e_C^* b - c(e_C^*). \tag{3}$$

The optimal litigation effort e_C^* is increasing in $\tilde{\theta}$, and so decreasing in $\tilde{\alpha}$. A lower probability of finding information and striking down the patent discourages the challenger's search activity. On the other hand, when engaging in a legal fight, the inventor always prefers a less intensive attack from the challenger (a lower e_C^*), while the challenger's payoff is decreasing in the probability of facing a strong patent $\tilde{\alpha}$, or, equivalently, increasing in $\tilde{\theta}$.

power where the challenger makes the offer, and a more general setting where the inventor has continuous types.

[19]Since we do not model the patent office as an active player, we ignore the issue of whether the patent office could commit to the examination effort announced at time 0. Even if there is commitment problem, our reasoning goes through as long as the examination effort is observable to other parties. In practice, the resources available to patent office and the incentive scheme offered to patent examiners are publicly observable, and so private parties can roughly figure out the examination effort to be exercised.

Denote $\underline{e}_C \equiv e_C(\underline{\theta})$ and $\bar{e}_C \equiv e_C(\bar{\theta})$ as the minimum and maximum possible optimal litigation efforts, corresponding to the lowest and highest possible θ, respectively. The optimal effort e_C^* lies in the interval $\in [\underline{e}_C, \bar{e}_C]$. Note that $\underline{e}_C > 0$ for $\underline{\theta} > 0$. It is easy to check that, given the same private litigation effort, the true inventor's expected payoff from litigation is strictly higher than that of the opportunistic player ($u_I(\underline{\theta}, e_C) > u_I(\bar{\theta}, e_C)$, $\forall e_C \in [\underline{e}_C, \bar{e}_C]$); and that, through its effect on e_C^* via $\tilde{\theta}$, an inventor's litigation payoff is increasing in the belief $\tilde{\alpha}$ (for both $\theta \in \{\underline{\theta}, \bar{\theta}\}$, $u_I(\theta, e_C^*)$ is increasing in $\tilde{\alpha}$).

Consider the bargaining stage. A settlement offer is a transfer between the inventor and challenger. If the inventor's settlement offer s is accepted by the challenger,[20] the inventor's and challenger's payoffs are $\pi - s$ and s, respectively. But if the offer is rejected and litigation ensues, the inventor receives an expected payoff $u_I(\theta, e_C^*)$ and the challenger receives $u_C(\tilde{\theta})$, where $\theta \in \{\underline{\theta}, \bar{\theta}\}$, and e_C^* as well as $\tilde{\theta}$ are determined by the challenger's belief in court $\tilde{\alpha}$. By $b < \pi$, the case is always settled under symmetric information: $\pi - u_C(\underline{\theta}) > u_I(\underline{\theta}, \underline{e}_C)$ and $\pi - u_C(\bar{\theta}) > u_I(\bar{\theta}, \bar{e}_C)$. The following lemma characterizes the case selection pattern in our model, i.e., which type of inventor is more likely to litigate.

Lemma 1. (Case selection). Consider the bargaining between the inventor and challenger.

- *There is no bargaining equilibrium in which the true inventor settles, but the opportunistic inventor litigates (even with only some probability).*
- *There is a bargaining equilibrium in which the opportunistic inventor litigates with a strict positive probability only when*

$$u_I(\bar{\theta}, \underline{e}_C) > \pi - u_C(\bar{\theta}). \tag{4}$$

Intuitively, when one party holds private information about her case quality (θ here), a stronger case (lower θ) makes a "tougher" player at the bargaining table and so a settlement deal is harder to reach. This result of "the innocent's curse" is fairly general and well-established in the literature of law and economics. In our context, it suggests that private enforcement cannot only be directed toward the "right target," that is, the weak patent; provoking private litigation at best improves patent quality at the expense of the true inventor.

[20] This offer s may depend on the type of the inventor. See the proof for more details.

To understand condition (4), the necessary condition when private enforcement can possibly eliminate the weak patent, note that $u_I(\bar{\theta}, \underline{e}_C)$ and $u_C(\bar{\theta})$ are the opportunistic inventor's and the challenger's highest possible payoff in litigation, respectively. The challenger is willing to accept a settlement payment $u_C(\bar{\theta})$ because this is his maximum possible payoff in court. The opportunistic inventor thus can guarantee herself a payoff $\pi - u_C(\bar{\theta})$ in a settlement. Because litigation gives the opportunistic inventor at most a payoff $u_I(\bar{\theta}, \underline{e}_C)$, when condition (4) fails, the opportunistic inventor prefers settlement to litigation.

When condition (4) fails, private enforcement cannot improve the patent quality, i.e., the probability that only the strong patent remains in the economy. When both types of inventor settle, private enforcement is inactive and the patent quality is solely determined by the patent office examination level. And when only the true inventor litigates, the weak patent won't be invalidated in court and so the patent quality is *lower* after private litigation.

Corollary 1. *When $u_I(\bar{\theta}, \underline{e}_C) \leq \pi - u_C(\bar{\theta})$, private enforcement doesn't improve the patent quality.*

Remark 1. ("Harassing" the true inventor). In the context of innovation, the "the innocent's curse," i.e., the true inventor's higher litigation risk, may translate into a higher probability of losing patent protection. This would be the case when the challenger litigates only against the true inventor, while settling the case with the opportunistic inventor.[21] When this happens, not only is a trued inventor "harassed" when trying to enforce her patent rights,[22] but private enforcement also reduces the true inventor's payoff and impairs R&D incentives without offsetting gains to raise the patent quality. ■

We offer two special cases before characterizing the equilibrium of the bargaining game.

[21]Other bargaining outcomes are considered in the proof of Proposition 1. We show that when $u_I(\underline{\theta}, \underline{e}_C) \geq \pi - u_C(\bar{\theta}) \geq u_I(\bar{\theta}, \underline{e}_C)$, there is a PBE where the true inventor litigates for sure and the opportunistic inventor settles for sure, with litigation efforts $\underline{e}_C$. In this equilibrium, the probability that the opportunistic inventor and true inventor receive patent rights are $1 - \bar{\theta} e_P$ and $(1 - \underline{\theta} e_P)(1 - \underline{\theta} \underline{e}_C)$, respectively. The opportunistic inventor has higher probability to survive and receive patent protection than the true inventor if $1 - \bar{\theta} e_P > (1 - \underline{\theta} e_P)(1 - \underline{\theta} \underline{e}_C) \Rightarrow \underline{\theta} \underline{e}_C (1 - \underline{\theta} e_P) > e_P(\bar{\theta} - \underline{\theta})$. It is more likely to be the case when e_P is small.

[22]The harassment hypothesis usually refers to invalidation challenges facing a patent-holder from potential infringers or other stake-holders. One possible litigation shown in our model is exactly this invalidation suit.

Example 1. (An ironclad strong patent). When the strong patent can never be invalidated, $\underline{\theta} = 0$, as in Franzoni (1999), there is some probability that an opportunistic inventor can still face litigation. This is confirmed by that fact that, under this case, $u_I(\bar{\theta}, \underline{e}_C) = \pi > \pi - u_C(\bar{\theta})$.

However, without invalidation risk the true inventor will never pay the challenger to settle the case. There is no equilibrium in which private bargaining always reaches a deal. Another outcome ruled out by this assumption is one in which the private challenger learns the inventor's true type and settles with the opportunistic player while litigating against the true inventor. By $\underline{\theta} = 0$ and so $\underline{e}_C = 0$, this equilibrium candidate is busted by the opportunistic inventor's attempt to mimic the good type.■

Example 2. (Inelastic private enforcement capacity). Suppose that $\underline{\theta} > 0$ but the challenger has inelastic litigation capacity, as in Meurer (1989). For simplicity, let us consider the extreme case of fixed and costless $e_C > 0$.[23] After this modification, the weak patent is entirely exempted from private enforcement. A fixed e_C renders $u_C(\bar{\theta}) = \bar{\theta} e_C b < \pi - u_I(\bar{\theta}, e_C) = \bar{\theta} e_C \pi$, which violates condition (4).[24] This confirms that the challenger's litigation effort decision is a key ingredient in our analysis. ■

From now on we will focus on bargaining equilibria where private litigation will be initiated against the weak patent with a strictly positive probability. This is justified by our interests in using private enforcement to raise patent quality. As shown in the proof, this class of equilibria satisfies the $D1$ criterion (Cho and Kreps, 1987).[25]

Suppose that the necessary condition (4) holds and so the weak patent can face an invalidation challenge in court. We say that the weak patent is fully (partially) exposed to private litigation if the opportunistic inventor is certain to engage in a court fight (with a probability, respectively). By Lemma 1, whenever the opportunistic inventor litigates, so does the true inventor. Denote $\hat{\alpha}$ as the patent quality, or the challenger's

[23]With costly but fixed effort, we need only that the challenger has a credible threat to incur the cost in a legal fight, e.g., by assuming a cost smaller than $\underline{\theta} e_C b$.

[24]Introducing litigation cost only strengthens this result.

[25]The $D1$ criterion constrains the weight the challenger can put on the opportunistic type when litigation is an off-path event. Roughly speaking, the true inventor would have more to gain than the opportunistic inventor in a legal fight, and so $D1$ requires the opportunistic inventor be fully deleted from the challenger's off-path belief.

In the proof, we also consider other bargaining outcomes such as where both types of inventor settle and there is no litigation, and where only the true inventor litigates. However, no PBE exists that fulfills the criterion $D1$ and implements the two outcomes. "Divinity," though, retains these bargaining outcomes (Bank and Sobel, 1987). As a weaker requirement it only requires that the challenger believe the true inventor plays the deviant move at least as often as the opportunistic inventor. The "passive belief," for example, is allowed under divinity but not under $D1$.

belief, at the beginning of the bargaining stage, and define $\hat{\theta} \equiv \hat{\alpha}\underline{\theta} + (1 - \hat{\alpha})\bar{\theta}$.

Proposition 1. *(Private enforcement). Suppose that condition (4) holds. The Perfect Bayesian Equilibria (henseforth, PBE) that survive D*1 *are*

- *Full exposure: When $u_I(\bar{\theta}, e_C^*(\hat{\theta})) \geq \pi - u_C(\bar{\theta})$, there is a PBE in which no settlement is reached at all, and the challenger exerts litigation effort $e_C^*(\hat{\theta})$; and*
- *partial exposure: if $u_I(\bar{\theta}, e_C^*(\hat{\theta})) < \pi - u_C(\bar{\theta}) < u_I(\bar{\theta}, \underline{e}_C)$, there is a PBE in which the opportunistic inventor litigates with probability $x^* \in (0, 1)$, the true inventor always litigates, and the challenger, with a belief α_x^* upon litigation, exerts a litigation effort $e_{C,x}^* < e_C^*(\hat{\theta})$, where $e_{C,x}^*$, x^*, and α_x^* are determined by*

$$u_I(\bar{\theta}, e_{C,x}^*) = \pi - u_C(\bar{\theta}), \;\; e_{C,x}^* = e_C^*\big(\alpha_x^*\underline{\theta} + (1 - \alpha_x^*)\bar{\theta}\big), \;\; \alpha_x^* = \frac{\hat{\alpha}}{\hat{\alpha} + (1 - \hat{\alpha})x^*}. \quad (5)$$

In the full exposure regime, the opportunistic inventor fully mimics the true inventor and litigates with probability one. The equilibrium litigation effort, $e_C^*(\hat{\theta})$, is determined by the initial belief $\hat{\alpha}$ at the bargaining stage. Because the challenger is willing to accept a settlement offer $u_C(\bar{\theta})$, this equilibrium requires patent quality $\hat{\alpha}$ be high enough, so that $\hat{\theta}$ and litigation effort $e_C^*(\hat{\alpha})$ low enough: $u_I(\bar{\theta}, e_C^*(\hat{\theta})) \geq \pi - u_C(\bar{\theta})$.

In the partial exposure regime, the opportunistic inventor plays a mixed strategy and litigates with probability $x^* \in (0, 1)$. Settlement, then, reveals the inventor's type and the challenger will only accept the offer $u_C(\bar{\theta})$. To be willing to play a mixed strategy, the opportunistic inventor must be indifferent between paying $u_C(\bar{\theta})$ to settle and litigating against the equilibrium effort $e_{C,x}^*$, i.e., $\pi - u_C(\bar{\theta}) = u_I(\bar{\theta}, e_{C,x}^*)$. This pins down the equilibrium litigation effort, which in turn has to be the challenger's optimal response to a belief α_x^*, i.e., $e_{C,x}^* = e_C^*(\alpha_x^*\underline{\theta} + (1 - \alpha_x^*\bar{\theta}))$. The equilibrium belief, α_x^*, is determined by both the initial belief $\hat{\alpha}$ and the probability that the opportunistic inventor plays litigation. The opportunistic inventor's equilibrium probability to litigate, x^*, is fixed according to condition (5).

4 Public vs. Private Enforcement: Substitutes or Complements?

Let us turn to the relationship between public and private enforcement. Again we assume condition (4) holds and consider bargaining equilibria under which the weak patent will be litigated with strictly positive probability.

Recall that $\theta^0 \equiv \alpha\underline{\theta} + (1-\alpha)\bar{\theta}$ is the belief of patent invalidity at the beginning of the game. After patent issuance, by observing the patent office examination effort e_P, the challenger adjusts his assessment of facing a strong patent to

$$\hat{\alpha} = \frac{\alpha(1-\underline{\theta}e_P)}{\alpha(1-\underline{\theta}e_P) + (1-\alpha)(1-\bar{\theta}e_P)} = \frac{\alpha(1-\underline{\theta}e_P)}{1-\theta^0 e_P}. \tag{6}$$

This can be seen as the quality of an issued patent, and is also the challenger's belief when settlement bargaining starts. Intuitively, a higher level of public enforcement e_P raises the patent quality:

$$\frac{\partial\hat{\alpha}}{\partial e_P} = \frac{\alpha(1-\alpha)(\bar{\theta}-\underline{\theta})}{(1-\theta^0 e_P)^2} > 0. \tag{7}$$

Due to this monotonic relationship, we can consider the impact of public enforcement e_P by directly looking at the effect of patent quality $\hat{\alpha}$ on private enforcement.

According to Proposition 1, whether the weak patent falls into the full or partial exposure regime depends crucially on patent quality $\hat{\alpha}$. As discussed above, the full exposure regime requires a high enough patent quality $\hat{\alpha}$, so that the equilibrium litigation effort $e_C^*(\hat{\theta})$ is low enough. Intuitively, the opportunistic inventor is willing to mix with the true inventor and litigate only when she expects a low litigation effort in court. This is more likely to be the case when the patent office exerts significant examination effort e_P and maintains high patent quality $\hat{\alpha}$. Furthermore, in this regime a marginal increase in public enforcement e_P will *reduce* private enforcement effort e_C, for a higher patent quality $\hat{\alpha}$ weakens the challenger's litigation effort. In this regime, public enforcement crowds out private enforcement.

The partial exposure regime, on the other hand, happens for low $\hat{\alpha}$. In this case, low patent quality, the result of low public enforcement level e_P, triggers a high litigation effort $e_C^*(\hat{\theta})$ from the challenger, should the opportunistic inventor fully mix with the true inventor to litigate in court. Relative to facing a very litigious challenger in court, with an expected payoff $u_I(\bar{\theta}, e_C^*(\hat{\theta}))$, the opportunistic inventor is willing to offer the challenger a payment $u_C(\bar{\theta})$ to settle the case. In equilibrium, the opportunistic inventor is indifferent between litigation and settlement and plays a mixed strategy as described in Proposition 1.

Unlike the previous case, the partial regime exhibits a *positive* relationship between public and private enforcement. By Proposition 1 the opportunistic inventor's litigation probability $x^* = [\hat{\alpha}(1-\alpha_x^*)]/[(1-\hat{\alpha})\alpha_x^*]$ is increasing in $\hat{\alpha}$. Together with the fact that the belief α_x^* and litigation effort $e_{C,x}^*$ are fixed in this case, the probability that the

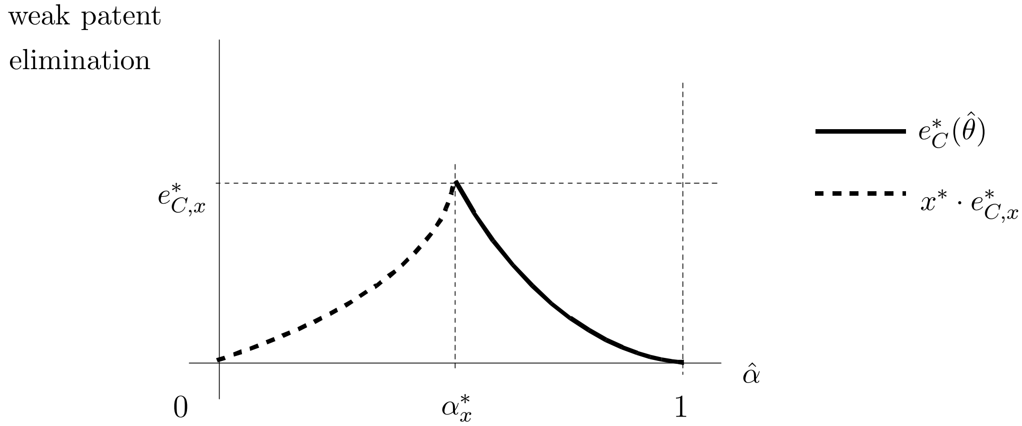

Figure 2: Patent quality and private enforcement

weak patent will be eliminated by private force, $x^* \cdot e^*_{C,x}$, is strictly increasing in e_P. In the partial exposure regime, public enforcement *crowds in* private enforcement.

The reason is that, referring to condition (5), under partial exposure the litigation effort $e^*_{C,x}$ is determined such that the opportunistic inventor is indifferent between paying $u_C(\bar{\theta})$ to settle the case and facing a challenge with effort $e^*_{C,x}$. On the other hand, to have $e^*_{C,x}$ as the best response, the challenger should hold a belief α^*_x when filing a challenge. Since a higher e_P will raise the quality of an issued patent $\hat{\alpha}$, the opportunistic inventor should litigate more (raise x^*) in order to maintain the challenger's belief at α^*_x.

Figure 2 summarizes the impact of patent quality $\hat{\alpha}$ on "weak patent elimination," which is defined as the probability that the weak patent will be eliminated in litigation. Since $\hat{\alpha}$ is strictly increasing in e_P, it also depicts the effect of public enforcement on private enforcement. As patent quality increases, we move from the partial exposure (the dashed line) to the full exposure regime (the solid line). A marginal increase in patent quality raises the probability of eliminating the weak patent in the former case, but not in the latter case. There is a non-monotonic relationship between weak patent elimination and patent quality.

Remark 2. (Empirical implications) It would be interesting to empirically test this non-monotonic relationship. A major challenge, however, is to find a good proxy for patent quality.[26] Given the general perception that the level of quality control varies

[26]A standard measure, forward citation, may capture more than the technological merit of a patent. Independent of the extent to which it fulfills the patentability standards, a patent would be cited more often

in different patent offices, one might want to conduct an international comparison. Doing so, nevertheless, requires overcoming the heterogenous litigation environments in different jurisdictions. Another way to test the theory might be to consider the impact of policy changes at the same patent office. For instance, in response to criticism of its lax quality control, the USPTO first introduced the "second pair of eyes" review in the area of business patents in 2001, and then extended it to other fields in 2004 and 2005 (Chen, 2009). Under this sysetm, a senior examiner and an examination panel are added to review the issuance decision. Assuming that this practice does improve patent quality, it might be desirable to check how it affects patent litigation. ||

Now let us derive some policy implications from this non-monotonic relationship. As discussed in the introduction, Lemley (2001)'s "rational ignorance" argument intends to substitute public enforcement with less costly private enforcement, *without jeopardizing overall patent quality.* However, our results suggest that when patent quality is already at the low end so that the partial exposure regime prevails, a further reduction of public enforcement effort e_P *will discourage* private enforcement toward the weak patent and decrease patent quality. When the economy suffers from low patent quality and raising patent quality is the primary concern, it would be desirable to improve patent office performance e_P. The overall cost of patent quality enforcement is certain to raise, but the key point is that in this case private enforcement is no substitute for public enforcement. Strengthening public enforcement not only contributes directly to raise patent quality, but also provides an indirect benefit of enhancing the involvement of private force in the quality control process.

On the other hand, the negative relationship at the full exposure regime leaves some scope for a "rationally ignorant" patent office. Reducing the patent office examination effort in this regime may decrease the overall examination cost without harming patent quality.

To check this possibility, recall that the patent office has a cost function $\gamma c(e_P)$, where $\gamma \geq 1$ and $c(\cdot)$ is the challenger's cost function. Define the total cost of patent examination as $C(e_P) \equiv \gamma c(e_P) + (1 - \theta^0 e_P)c(e^*_C(\hat{\theta}))$. Also define the overall level of examination a patent application is expected to receive as $e_P + e^*_C$, for under this regime, a patent applicant with θ expects rejection with probability $1 - (1 - \theta e_P)(1 - \theta e^*_C) = \theta(e_P + e^*_C) - \theta^2 e_P e^*_C \simeq \theta(e_P + e^*_C)$. We are concerned with when a marginal decrease in e_P will reduce the total cost $C(e_P)$ without reducing the overall examination standard

when it delivers more economic values or plays a more important role in a firm's R&D strategy.

$e_P + e_C^*$, i.e., the simultaneous satisfaction of $d(e_P + e_C^*)/de_P \leq 0$ and $dC(e_P)/de_P > 0$. The next proposition presents conditions when they will hold, and summarizes the main results in this section.

Proposition 2. *(Patent quality and patent office). Assume condition (4) holds so that the weak patent may be litigated in court.*

- *(Partial exposure) When patent quality is low enough, $\hat{\alpha} < \alpha_x^*$, the probability of eliminating the weak patent through private effort, $x^* \cdot e_{C,x}^*$, is increasing in e_P. In this regime, reducing public enforcement e_P deteriorates patent quality.*
- *(Full exposure) When patent quality is high enough, $\hat{\alpha} \geq \alpha_x^*$, a higher level of public enforcement e_P reduces the probability of eliminating the weak patent through litigation $e_C^*(\hat{\theta})$.*

  Consider a marginal decrease in e_P in this regime:

  - *it will not weaken the overall examination standard if and only if*

$$\frac{de_C^*(\hat{\theta})}{d\hat{\alpha}} \frac{\partial \hat{\alpha}}{\partial e_P} \leq -1 \quad \Rightarrow \quad \frac{\alpha(1-\alpha)(\bar{\theta} - \underline{\theta})^2 b}{c''(e_C^*)(1 - \theta^0 e_P)^2} \geq 1; \tag{8}$$

  - *it will reduce the total examination cost if and only if*

$$\gamma > \frac{1}{c'(e_P)} \left[\theta^0 c(e_C^*(\hat{\theta})) - (1 - \theta^0 e_P) c'(e_C^*(\hat{\theta})) \frac{de_C^*(\hat{\theta})}{d\hat{\alpha}} \frac{\partial \hat{\alpha}}{\partial e_P} \right]. \tag{9}$$

  When both conditions hold, the rational ignorance hypothesis is supported.

Not surprisingly, to justify a not-so-excellent patent office, condition (9) requires significant cost advantage of private enforcement, i.e., γ should be large enough. For the overall examination standard, a sufficient condition for condition (8) to hold is: $\forall e_C$, $\alpha(1-\alpha)(\bar{\theta} - \underline{\theta})^2 b \geq c''(e_C)$. This comes from the fact that the private sector's response should be large enough in order to compensate for a more lax public quality control. Among others, it requires a "less curved" cost function, i.e., c'' small enough, as $\partial e_C^*(\hat{\theta})/\partial \hat{\theta} = b/c''(e_C^*)$.

Remark 3. (R&D incentives). So far we've ignored the true inventor's R&D incentives. If this concern is introduced, in the presence of a "type II error," $\underline{\theta} > 0$, the patent office may want to constrain its examination effort e_P. Under the partial exposure regime, this can be done by reducing e_P. But in the full exposure regime, a reduction in e_P causes e_C to increase, and the overall enforcement level decreases if and only if condition (8) fails.

However, this analysis is entirely based on the adverse selection assumption on the inventor's side. Instead, we may take a moral hazard view where a true inventor is given both the opportunities of producing genuine innovations and patenting public domain technologies. In this case, a higher overall examination standard will serve as a more powerful "stick" to push the inventor away from the temptation of opportunistic patenting. This provides another channel to improve patent quality.[27] ■

Remark 4. (Application fees). Let us erase the limited liability protection and allow a negative return for an inventor, and so application fees can be added as a policy tool. When the patent office imposes fees on patent applicants, this ideally may deter the opportunistic inventor from seeking patent protection. In general, a more effective way to achieve this goal is to condition the pecuniary punishment on the examination outcome, e.g., upon the rejection of a patent application or invalidation of an issued patent in court. However, a fine after invalidation is arguably under the discretion of the court, and an applicant, especially a "short-run player," might simply run away when her application is rejected by the patent office. Instead, we consider a uniform application fee f for all patent applications. Nevertheless, our main result is not affected by the fee structure under study.

Suppose that an application fee f fully deters the opportunistic inventor from applying for a patent, but not the true inventor. When this is true, at the bargaining stage the challenger holds belief that $\hat{\alpha} = 1$, and symmetric information prevents bargaining breakdown. In our model, a fully deterrent application fee mutes entirely private enforcement. When the inventor holds the bargaining power,[28] it suffices to pay $u_C(\underline{\theta})$ to settle the case, and a deterrent fee f should satisfy

$$(1 - \bar{\theta} e_P)\pi - u_C(\underline{\theta}) < f \leq (1 - \underline{\theta} e_P)\pi - u_C(\underline{\theta}). \tag{10}$$

Since this condition will not hold when $e_P = 0$, a deterrent application fee cannot substitute for patent office examination. Furthermore, to preserve the true inventor's

[27]To see this, suppose that for the true inventor, there is a choice between doing innovation (at a cost $K > 0$, with patent validity $\underline{\theta}$) and engaging in opportunistic patenting (at no cost, with patent validity $\bar{\theta}$). Assume that the weak patent is not settled for sure. The incentive compatibility constraint to push the inventor to do R&D is $(1 - \underline{\theta} e_P)(1 - \underline{\theta} e_C)\pi - K \geq (1 - \bar{\theta} e_P)(1 - \bar{\theta} e_C)\pi$, where e_P and e_C are the prevailing public and private enforcement efforts, respectively. (Note that in the partial exposure regime, the weak patent-holder is indifferent between litigation and settlement.) When $e_P \cdot e_C \simeq 0$, the constraint becomes $(\bar{\theta} - \underline{\theta})(e_P + e_C)\pi \geq K$, which is more likely to satisfy when the overall quality control level $e_P + e_C$ is higher.

[28]The distribution of bargaining power is not crucial to this result. It only changes the level of f to deter opportunistic patenting, for the patent-holder's payoffs from fully settling the case depend on who makes the offer.

returns, the patent office should set f as small as possible, without losing its deterrent power. Let $f^D = (1-\bar{\theta}e_P)\pi - u_C(\underline{\theta}) + \epsilon$, with $\epsilon > 0$ but small. Since f^D is decreasing in e_P, the true inventor's payoff, $(1-\underline{\theta}e_P)\pi - u_C(\underline{\theta}) - f^D = (\bar{\theta}-\underline{\theta})e_P\pi - \epsilon$, is increasing in e_P.

Proposition 3. *(Application fees). An application fee that fully deters opportunistic patenting crowds out private enforcement but cannot substitute for public enforcement. A higher patent office examination level e_P reduces the necessary fee. When the application fee is set at the minimal necessary level f^D, the true inventor's payoff, and so the R&D incentive, is increasing in e_P.* ■

5 Concluding Remarks

The limitation of private enforcement emphasized in this paper, namely the settlement bias toward weak patents, would persist despite the private challenger's information and cost advantages. It highlights the importance of a decent patent office. Accordingly, future works and reform efforts should focus on how to improve the performance of the patent office in order to "get things right" in the first place. The agency problem and task allocations within the patent office are additional topics in our research agenda.[29]

In this aspect, our analysis sheds some light on the design of incentive payments for patent examiners. One difficulty in constructing this incentive scheme is finding a proper index of examiners' efforts.[30] A straightforward and somewhat "naive" application of incentive theory might suggest the use of court rulings as a measure of performance: A patent examiner would be punished if a patent issued by her is later invalidated in court. Several practical issues reduce the usefulness of this measure. For instance, the rare occurrence of patent disputes and the strong tendency toward settlement; upon dispute, the long delay from patent issuance to the final court judgment; and, at least in the United States, a significant portion of patent examiners who choose a career path in the private sector after a few years' experience in the patent

[29] Merges (1999) argues that the U.S. patent examiners are given incentives to approve, but not reject patent applications. Based on surveys of patent examiners in the USPTO and European patent office, respectively, Cockburn *et al.* (2003), Friebel *et al.* (2006) provide useful insights about the process and feature of patent examination, the internal functioning and organization of patent offices, and examiners' incentives, *etc.*.

[30] This issue is closely related to finding a satisfactory measure of patent quality discussed in Remark 2. Langinier and Marcoul (2009) and Schuett (2009) are two recent efforts devoted to patent examiners' incentive problems.

office. Our analysis points out another limitation: the information content of a court ruling may be distorted by private bargaining. In particular, a positive relationship between public and private enforcement in the partial exposure regime suggests that a higher effort by the patent examiner may result in more patents being litigated and invalidated in court. It would then be undesirable to punish the examiner upon a successful post-grant court challenge.

Another direction for future research is to extend the analysis to more complex environments. This would allow us to check whether our results are robust to other bargaining paradigms.

- Two-sided asymmetric information: When enforcing her patent rights, the patent-holder may have less information than a potential infringer about the latter's infringement activities (e.g. the true infringement probability or damage). This introduces another asymmetric information element into the model. Assume that the two pieces of private information are independently distributed. Since, *ceteris paribus*, a patent-holder would be tougher at the bargaining table when having a strong patent, this modification should have little impact on the case selection pattern in Lemma 1. It would be interesting to see whether and how our second main result, namely, the non-monotonic relationship between public and private enforcement (Figure 2), would be affected in a bargaining setting characterized by two-sided asymmetric information.

- Divergent expectations: Suppose that the hurdle of settlement is the two disputants' different assessments of case quality (patent validity here), and that they agree to disagree regarding each other's assessment (Priest and Klein, 1984). A general result from this approach is that bargaining breaks down when the true case quality falls in the "middle range," since this is the case most likely to lead to extreme expectations. In our context, it means that neither patents with very low or very high validity will be litigated. Therefore, with some modification our first result still holds: Private enforcement will only attack mediocre patents. As to the effect of intensifying public enforcement, we need to figure out how the patent office's efforts affect the discrepancy between the parties' case assessments and true case quality. This would require a model of the expectation-generating process (e.g., whether and how the noise comes from the litigation process as well as court decision-making), and how it is related to the patent office examination.

- Asymmetric stakes: Alternatively, we can consider a more involved industry or innovation structure. For instance, in a cumulative innovation process a potential challenger may be an inventor or patent-holder from another generation of technology development. Two twists may then be present: multiple contacts (the two patent-holders may threat to initiate a litigation war to invalidate each other's patent), and reverse case stake (i.e., $b > \pi$, the potential challenger has a larger stake to invalidate the patent than the patentee's monopoly profit). Since quite a few high-tech industries exhibit this feature, it would further advance our knowledge of the case selection pattern, and more generally, help us understand how to improve patent quality through the joint efforts of public and private enforcement in specific sectors.

Appendix

A Proofs

□ Lemma 1

Proof. Consider an equilibrium in which the true inventor settles (with some probability) but the opportunistic inventor always litigates. Let s' be (one of) the true inventor's equilibrium settlement payment(s), which may be adopted for some probability, and $e'_C > 0$ be (one of) the litigation efforts facing the opportunistic inventor. When the true inventor prefers settlement and paying s' than litigation against an effort e'_C, $\pi - s' \geq u_I(\underline{\theta}, e'_C) > u_I(\bar{\theta}, e'_C)$, the opportunistic inventor has incentives to deviate to s' and settle.

The reason that condition (4) is the necessary condition for the opportunity type to litigate is stated in the main context. *Q.E.D.*

The following lemma is convenient to subsequent analysis.

Lemma 2. (Off-path belief selection and full settlement). Consider a PBE where no litigation occurs, and denote s as the equilibrium settlement payment from the inventor to the challenger. If this equilibrium fulfills the criterion $D1$ (divinity), it must be supported by off-path beliefs $\tilde{\alpha} = Pr(\underline{\theta}|\tilde{s})$ such that for $\tilde{s} < s$, $\tilde{\alpha} = 1$ ($\tilde{\alpha} \geq \hat{\alpha}$, respectively).

Proof. To use $D1$ or divinity to eliminate or constrain the weight on the opportunistic type when observing a deviation $\tilde{s} < s$, we show that whenever a (mixed strategy) best response of the challenger to $\tilde{s}$ makes the opportunistic inventor (weakly) better off than under the equilibrium, the same response must give the true inventor a strictly higher payoff than the equilibrium payoff.

Let s be the equilibrium payment from the inventor to the challenger in a PBE where no litigation occurs. Note that there can be only one such payment, otherwise the player making the offer will deviate to the payment that serves best his/her interests without intriguing law suits. The inventor's equilibrium payoff is $\pi - s$, regardless of her type. Consider the challenger's belief upon an off-path offer $\tilde{s} < s$.

Suppose that the inventor makes the offer. If the challenger observes $\tilde{s} < s$, denote his mixed strategy best response as $(\tilde{\phi}, \tilde{e}_C)$ and belief as $\tilde{\alpha}$, where $\tilde{\phi}$ is the probability to accept the offer and $\tilde{e}_C = e_C^*(\tilde{\theta})$ the litigation effort when rejecting the offer, given $\tilde{\theta} = \tilde{\alpha}\underline{\theta} + (1 - \tilde{\alpha})\bar{\theta}$. The inventor's payoff from deviating to $\tilde{s}$ is therefore $\tilde{\phi}(\pi - \tilde{s}) + (1 - \tilde{\phi})u_I(\theta, \tilde{e}_C)$, $\theta \in \{\underline{\theta}, \bar{\theta}\}$. By the shape of $c(\cdot)$, the challenger doesn't mix among different levels of e_C.

Since $\pi - \tilde{s} > \pi - s$, when $\tilde{\phi} = 1$ both types of inventor strictly prefer the deviation. When $\tilde{\phi} = 0$, for any $\tilde{e}_C > 0$, $u_I(\underline{\theta}, \tilde{e}_C) > u_I(\bar{\theta}, \tilde{e}_C)$ and so whenever the opportunistic inventor is (weakly) better off by deviating to $\tilde{s}$, the true inventor strictly prefers doing so. The same holds when $\tilde{\phi} \in (0, 1)$.

When the challenger makes the offer, to support this equilibrium the inventor must reject $\tilde{s}$ and this deviant offer must lead to litigation. Previous argument guarantees that if the opportunistic inventor weakly prefers to deviate under some $\tilde{e}_C$, the true inventor must strictly prefer doing so. *Q.E.D.*

□ Proposition 1

Proof. For similar reason stated in the proof of Lemma 2, there can be at most one equilibrium litigation effort e_C.

⋄ Full exposure: Along the equilibrium path, both types of inventor propose a settlement offer $s < u_C(\hat{\theta})$ and the challenger rejects this offer while maintaining belief at $\hat{\theta}$, with litigation effort $e_C^*(\hat{\theta})$. The inventor's equilibrium payoff is $u_I(\theta, e_C^*(\hat{\theta}))$, $\theta \in \{\underline{\theta}, \bar{\theta}\}$. To prevent deviation, (i) since the challenger will agree to settle with a payment $u_C(\bar{\theta})$, the opportunistic inventor should prefer litigation to settlement for sure, $u_I(\bar{\theta}, e_C^*(\hat{\theta})) \geq \pi - u_C(\bar{\theta})$; and (ii) for other deviations $\tilde{s} < u_C(\bar{\theta})$, the challenger needs to reject $\tilde{s}$ and litigates with $\tilde{e}_C \geq e_C^*(\hat{\theta})$, to be supported by off-path belief $\tilde{\alpha} \leq \hat{\alpha}$.

$\diamond$ Partial exposure: This is a semi-pooling equilibrium where the opportunistic inventor mixes with the true inventor and litigate with probability $x^* \in (0, 1)$. The challenger's equilibrium belief upon litigation therefore is α_x^* specified in condition (5), which in turn determines $e_{C,x}^*$. Since only the opportunistic inventor settles, the settlement offer $s^* = u_C(\bar{\theta})$, and she is willing to play mixed strategy iff $\pi - u_C(\bar{\theta}) = u_I(\bar{\theta}, e_{C,x}^*)$. This guarantees that the true inventor won't deviate to offer s^*. By $\alpha_x^* \in (\hat{\alpha}, 1)$ and so $e_{C,x}^* \in (\underline{e}_C, e_C^*(\hat{\theta}))$, we can find such $e_{C,x}^*$ iff $\pi - u_C(\bar{\theta}) \in (u_I(\bar{\theta}, e_C^*(\hat{\theta})), u_I(\bar{\theta}, \underline{e}_C))$. To support this equilibrium, the challenger should reject any deviant offer $\tilde{s} < u_C(\bar{\theta})$ and litigate with $\tilde{e}_C \geq e_{C,x}^*$. In other words, the challenger should put enough weight on the opportunistic inventor upon receiving $\tilde{s} < u_C(\bar{\theta})$.

To show that both equilibria survive $D1$, it suffices to show that the opportunistic inventor cannot be deleted in the challenger's off-path beliefs. Since the inventor's equilibrium payoff is $u_I(\theta, e_C)$, depending on the inventor's type and the prevailing e_C in each equilibrium, observing a deviation offer, the challenger's response of rejection and litigation with the equilibrium efforts level makes both types of inventor indifferent from deviation or not. And by $u_I(\bar{\theta}, e_C) < u_I(\underline{\theta}, e_C)$, whenever the challenger's acceptance of a deviant offer makes the true inventor weakly better-off by deviating, the opportunistic inventor strictly prefers that deviation. Hence $D1$ cannot rule out the opportunistic type.

For other bargaining outcomes:

$\diamond$ No litigation: The minimal offer to settle with both types of inventor is $u_C(\hat{\theta})$. Let it be an equilibrium payment. To support this equilibrium, let the challenger accept any deviant offers larger than $u_C(\hat{\theta})$ with, say, "passive belief" $\hat{\theta}$. When facing a smaller offer, the challenger should reject it and exert litigation effort $\tilde{e}_C$ such that $u_I(\underline{\theta}, \tilde{e}_C) \leq \pi - u_C(\hat{\theta})$. But by Lemma 2, $D1$ requires that the challenger believe that such an offer comes from the good type for sure, which in turn requires the challenger to accept any offer in $(u_C(\underline{\theta}), u_C(\hat{\theta}))$. Therefore no PBE fulfilling $D1$ can implement this outcome. On the other hand, since the passive belief is allowed under divinity, and u_I is decreasing in e_C, no litigation can be implemented by a PBE satisfying divinity if $u_I(\underline{\theta}, e_C^*(\hat{\theta})) \leq \pi - u_C(\hat{\theta})$.

$\diamond$ Only the true inventor litigates: First consider a full separating equilibrium such that the true inventor always litigates while the opportunistic inventor always settles. In this case, the opportunistic inventor's equilibrium offer is $u_C(\bar{\theta})$, and the true inventor litigates against an effort $\underline{e}_C$. Neither type will deviate to play the other's equilibrium strategy when $u_I(\underline{\theta}, \underline{e}_C) \geq \pi - u_C(\bar{\theta}) \geq u_C(\bar{\theta}, \underline{e}_C)$. No inventor would offer higher than $u_C(\bar{\theta})$ to settle the case. To support the equilibrium, the challenger has to reject a deviant offer $\tilde{s} < u_C(\bar{\theta})$ and litigating with $\tilde{e}_C \geq \underline{e}_C$. Since the inventor can be sure to face the minimal effort $\underline{e}_C$ by proposing the true inventor's offer (it could be an empty offer), no patent-holder has incentives to deviate to any other offers strictly smaller than $u_C(\underline{\theta})$.

Consider a deviant offer $\tilde{s} \in [u_C(\underline{\theta}), u_C(\bar{\theta}))$. To reject this offer, the challenger should put enough weight on the opportunistic type, i.e., $\tilde{\theta}$ so high that $\tilde{s} < u_C(\tilde{\theta})$. We show that for $\tilde{s}$ small enough, $D1$ would require $Pr(\underline{\theta}|\tilde{s}) = 1$ and so this outcome cannot be supported as an equilibrium outcome. Relaxing the requirement to divinity, this outcome is possible only when $\hat{\alpha}$ small enough. Denote $(\tilde{\phi}, \tilde{e}_C)$ as the challenger's optimal response to $\tilde{s}$, which is rationalized by belief $\tilde{\alpha}$.

If $\tilde{s} \in [\pi - u_I(\underline{\theta}, \underline{e}_C), u_C(\bar{\theta}))$, the challenger's response $\tilde{\phi} = 1$ makes the opportunistic inventor, but not the true inventor, strictly better off, relative to their equilibrium payoffs. $D1$ and divinity cannot constrain $\tilde{\theta}$. For $\tilde{s} \in [u_C(\underline{\theta}), \pi - u_I(\underline{\theta}, \underline{e}_C))$, (i) if $\tilde{\phi} = 1$, both types of inventor strictly prefer $\tilde{s}$ than their equilibrium strategy; (ii) if $\tilde{\phi} = 0$ and $\pi - u_C(\bar{\theta}) > u_I(\bar{\theta}, \underline{e}_C)$, whatever $\tilde{e}_C$, this response cannot make the good (opportunistic) inventor strictly (weakly, respectively) better off; and (iii) if $\tilde{\phi} \in (0,1)$, then for the challenger to take mixed strategy response, $\tilde{s} = u_C(\tilde{\theta})$ and $\tilde{e}_C = e^*_C(\tilde{\theta})$. The opportunistic inventor weakly prefers to deviate if

$$\tilde{\phi}(\pi - \tilde{s}) + (1 - \tilde{\phi}) u_I(\bar{\theta}, \tilde{e}_C) \geq \pi - u_C(\bar{\theta}) \Rightarrow \tilde{\phi} \geq \bar{\phi} \equiv \frac{\pi - u_C(\bar{\theta}) - u_I(\bar{\theta}, \tilde{e}_C)}{\pi - u_C(\tilde{\theta}) - u_I(\bar{\theta}, \tilde{e}_C)}; \quad \text{(A.1)}$$

and the true inventor strictly prefers to deviate if

$$\tilde{\phi}(\pi - \tilde{s}) + (1 - \tilde{\phi}) u_I(\underline{\theta}, \tilde{e}_C) > u_I(\underline{\theta}, \underline{e}_C) \quad \text{(A.2)}$$

$$\Rightarrow \quad \pi - u_C(\tilde{\theta}) > u_I(\underline{\theta}, \tilde{e}_C) \;\text{ and }\; \tilde{\phi} > \underline{\phi} \equiv \frac{u_I(\underline{\theta}, \underline{e}_C) - u_I(\underline{\theta}, \tilde{e}_C)}{\pi - u_C(\tilde{\theta}) - u_I(\underline{\theta}, \tilde{e}_C)}. \quad \text{(A.3)}$$

$D1$ and divinity have no bite for those $\tilde{s}$ such that $\pi - u_C(\tilde{\theta}) \leq u_I(\underline{\theta}, \tilde{e}_C)$. But this won't be the case for all $\tilde{\theta}$, for $\pi > u_I(\underline{\theta}, \underline{e}_C) + u_C(\underline{\theta})$ as $\tilde{\theta} \to \underline{\theta}$ (as $\tilde{s} \to u_C(\underline{\theta})$). Define $\tilde{S} \equiv \{\tilde{s} : u_I(\underline{\theta}, \tilde{e}_C) + u_C(\tilde{\theta}) < \pi, \bar{\phi} > \underline{\phi}\}$. $\tilde{S} \neq \emptyset$ since, as $\tilde{s} \to u_C(\underline{\theta})$,

$$\bar{\phi} \to \frac{\pi - u_C(\bar{\theta}) - u_I(\bar{\theta}, \underline{e}_C)}{\pi - u_C(\underline{\theta}) - u_I(\bar{\theta}, \underline{e}_C)} > 0, \quad \text{but} \quad \underline{\phi} \to \frac{u_I(\underline{\theta}, \underline{e}_C) - u_I(\underline{\theta}, \underline{e}_C)}{\pi - u_C(\underline{\theta}) - u_I(\underline{\theta}, \underline{e}_C)} = 0. \quad \text{(A.4)}$$

For all $\tilde{s} \in \tilde{S}$, the set of the challenger's strictly mixed strategy best responses that makes the true inventor strictly prefer to deviate is strictly larger than the set that makes the opportunistic inventor weakly prefer to deviate. Therefore, for any $s' \in S' \equiv \tilde{S} \cap [u_C(\underline{\theta}), \pi - u_I(\underline{\theta}, \underline{e}_C))$, $D1$ requires the challenger to hold belief $\theta' = \underline{\theta}$, and divinity requires a belief $\theta' \leq \hat{\theta}$. Imposing $D1$ then eliminates this full separating equilibrium, as the challenger should accept the offer $u_C(\underline{\theta})$. And divinity will bust the equilibrium when $\hat{\alpha}$ is so large, and $\hat{\theta}$ so small that $u_C(\hat{\theta}) \leq s'$ for some $s' \in S'$, since the challenger needs to reject s' with some θ' such that $u_C(\theta') > s'$.

Lastly, suppose $\pi - u_C(\bar{\theta}) = u_I(\bar{\theta}, \underline{e}_C)$. In this case $D1$ and divinity have no bite for (i) when $\tilde{s} = u_C(\underline{\theta})$, the challenger's response $\tilde{\phi} = 0$ and $\tilde{e}_C = \underline{e}_C$ makes both types of inventor indifferent between deviation or not; and (ii) when $\tilde{s} \in (u_C(\underline{\theta}), \pi - u_I(\underline{\theta}, \underline{e}_C))$,

$$\bar{\phi} = \frac{u_I(\bar{\theta}, \underline{e}_C) - u_I(\bar{\theta}, \tilde{e}_C)}{\pi - u_C(\hat{\theta}) - u_I(\bar{\theta}, \tilde{e}_C)} = \frac{\bar{\theta}(\tilde{e}_C - \underline{e}_C)\pi}{\bar{\theta}\tilde{e}_C\pi - u_C(\tilde{\theta})} < \underline{\phi} = \frac{\underline{\theta}(\tilde{e}_C - \underline{e}_C)\pi}{\underline{\theta}\tilde{e}_C\pi - u_C(\tilde{\theta})}, \tag{A.5}$$

even when $\pi - u_C(\hat{\theta}) - u_I(\underline{\theta}, \tilde{e}_C) > 0$.

◇ The true inventor plays mixed strategies: Lastly, if the true inventor plays the mixed strategy, denote y^* as her equilibrium probability to settle. The challenger's belief upon settlement then is α_y^*, with $\theta_y^* = \alpha_y^*\underline{\theta} + (1 - \alpha_y^*)\bar{\theta}$, and the equilibrium settlement offer $s^* = u_C(\theta_y^*)$, such that

$$u_I(\underline{\theta}, \underline{e}_C) = \pi - u_C(\theta_y^*) \text{ and } \alpha_y^* = \frac{\hat{\alpha}y^*}{\hat{\alpha}y^* + 1 - \hat{\alpha}}. \tag{A.6}$$

Since only the true inventor litigates, the equilibrium litigation effort is $\underline{e}_C$. The true inventor is willing to play a mixed strategy iff $u_I(\underline{\theta}, \underline{e}_C) = \pi - u_C(\theta_y^*)$, which leaves the opportunistic inventor no incentives to deviate and litigate. Since $\alpha_y^* \in (0, \hat{\alpha})$ and so $u_C(\theta_y^*) \in (u_C(\hat{\theta}), u_C(\bar{\theta}))$, this equilibrium requires $u_I(\underline{\theta}, \underline{e}_C) \in (\pi - u_C(\bar{\theta}), \pi - u_C(\hat{\theta}))$. Note that any deviant offer leading to litigation won't disturb this equilibrium, for the inventor's equilibrium payoff is $\pi - u_C(\theta_y^*) = u_I(\underline{\theta}, \underline{e}_C) > u_I(\bar{\theta}, \underline{e}_C)$. We then check whether there is belief satisfying divinity and inducing the challenger's rejection of a deviant offer $\tilde{s} \in [u_C(\bar{\theta}), u_C(\theta_y^*))$. Since $\alpha_y^* < \hat{\alpha}$ and so $u_C(\hat{\theta}) < u_C(\theta_y^*)$, (i) for $\tilde{s} \in [u_C(\underline{\theta}), u_C(\hat{\theta}))$, whether divinity can trim the challenger's off-path belief, upon deviation we can use the passive belief $\hat{\theta}$ to justify the challenger's rejection; and (ii) for $\tilde{s} \in [u_C(\hat{\theta}), u_C(\theta_y^*))$, it can be rejected only with belief $\tilde{\theta}$ such that $u_C(\tilde{\theta}) > \tilde{s} \geq u_C(\hat{\theta})$, and so to have $\tilde{\theta} > \hat{\theta}$ the weight on the opportunistic inventor should not be constrained by divinity. The challenger's accepting $\tilde{s}$ makes both types of inventor strictly better off; his rejection, together with litigation effort strictly higher than $\underline{e}_C$ makes the inventor worse off. But if the challenger plays a mixed strategy composed of $\tilde{\phi} \in (0, 1)$

and $\tilde{e}_C$, since the inventor's equilibrium payoff doesn't not depend on her type, and

$$\tilde{\phi}(\pi - \tilde{s}) + (1 - \tilde{\phi})u_I(\underline{\theta}, \tilde{e}_C) > \tilde{\phi}(\pi - \tilde{s}) + (1 - \tilde{\phi})u_I(\bar{\theta}, \tilde{e}_C), \tag{A.7}$$

whenever the opportunistic inventor weakly prefers to deviate, the true inventor strictly prefers to do so. For this range of $\tilde{s}$, divinity then requires off-path belief $\tilde{\theta} \leq \hat{\theta}$, and so this equilibrium cannot survive divinity. *Q.E.D.*

□ **Proposition 2**

Proof. The necessary and sufficient conditions come directly from $d[e_P + e_C^*(\hat{\theta})]/de_P \leq 0$ and $dC(e_P)/de_P > 0$: A marginal change in e_P causes a change in the overall standard by

$$\frac{d[e_P + e_C^*(\hat{\theta})]}{de_P} = 1 + \frac{de_C^*(\hat{\theta})}{d\hat{\alpha}}\frac{\partial\hat{\alpha}}{\partial e_P} = 1 - \frac{(\bar{\theta} - \underline{\theta})b}{c''(e_C^*)}\frac{\alpha(1-\alpha)(\bar{\theta} - \underline{\theta})}{(1 - \theta^0 e_P)^2}, \tag{A.8}$$

and a change in the total cost by

$$\frac{dC(e_P)}{de_P} = \gamma c'(e_P) - \theta^0 c(e_C^*(\hat{\theta})) + (1 - \theta^0 e_P)c'(e_C^*(\hat{\theta}))\frac{de_C^*(\hat{\theta})}{d\hat{\alpha}}\frac{\partial\hat{\alpha}}{\partial e_P}. \tag{A.9}$$

The sufficient condition of no lower examination standard is obtained by setting $e_P = 0$ in condition (8), and the necessary condition of no larger cost is obtained by inserting $(de_C^*/d\hat{\alpha})(\partial\hat{\alpha}/\partial e_P) \leq -1$ into $dC(e_P)/de_P > 0$. *Q.E.D.*

B Pre-grant Challenges

In this appendix, we consider a pre-grant challenge system. Suppose that after receiving a patent application but before starting its examination process (time 1.5 in Figure 3), the patent office publishes the application and allows third parties to challenge it (or to submit information concerning its patentability).[31]

Introducing a pre-grant challenge procedure allows the patent office to set different examination levels according to an application's history. Let e_P^c be the examination effort exerted on an application that has survived private challenges, and e_P^n on one which has not yet been challenged. Intuitively, the patent office should set $e_P^c \leq e_P^n$. In

[31] Early publication of patent applications (18 months after filing) has been widely adopted in Japan and Europe; the U.S. has the same procedure but allows an applicant to opt out. About the pre-grant challenge, the 2007 Patent Reform Act in the U.S. introduces a procedure permitting third parties to submit relevant information before the issuance of a patent.

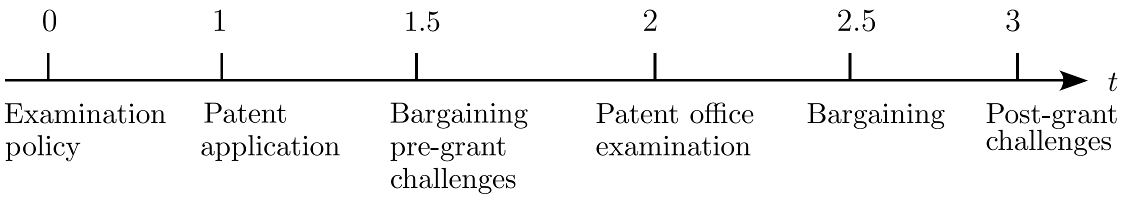

Figure 3: Timing with pre-grant challenges

addition to the fact that private enforcement efforts perform as a "certificate" about the validity of an application, one might cite the case selection pattern of Lemma 1 as another support of such a policy.

However, under such a policy, an applicant may try to circumvent the high effort e_P^n by arranging a "fake" challenge, in particular when the patent office is unable to verify the challenger's effort level. That is, whether the challenger only initiates a nominal challenge procedure without any serious effort to strike down the application. Besides, we further argue that (i) the "direction" of case selection may be reversed at the pre-grant challenge stage. Contrary to the previous result, there may exist an equilibrium where only the true inventor settles at the pre-challenge bargaining; and (ii) when the challenger does intend to initiate a challenge, and both pre- and post-grant challenges are available, he may want to wait and file a private challenge only after the failure of the patent office.[32] That is, the challenger may want to free ride on the patent office's efforts.

For the first point, suppose that the challenger can only initiate a challenge at the pre-grant stage, and that the inventor's settlement payment comes from the monopoly rent and so is paid only when the patent is issued. (This is the case when the inventor is protected by limited liability.) Recall that the challenger cannot commit to e_C in an agreement, and his initial belief of patent (application) quality is α. We derive conditions under which there is a separating equilibrium where only the true inventor settles. A necessary condition is both $\underline{\theta}$ and $e_P^c > 0$. The former is simply due to the fact that a true inventor with $\underline{\theta} = 0$ will never pay anything to settle. The latter can be justified in that the patent office doesn't "outsource" the examination task entirely to private parties, or doesn't "rubber stamp" the issuance of a patent following private efforts. Even if an application survives private challenges, the patent office still does its own work.

[32]Of course, this is more likely the case when costs accrued to challengers are not so different for the post- and pre-grant challenge procedures.

Intuitively, when the patent office sets different examination levels according to the challenge history, the inventor will take this into account when making settlement decisions. Consider if $e_P^n >> e_P^c$, that is, if an unchallenged application will receive much more attention from the patent office than an application surviving private challenges. This gives an applicant incentives not to settle with a private challenger in order to avoid stringent public scrutiny. But the magnitude of this effect depends on the true quality of the invention θ. For instance, when $\underline{\theta}$ is very close to zero, even $e_P^n \simeq 1$ won't harm the true inventor too much. This may reverse the case selection pattern at the pre-grant challenge stage: Only the true inventor settles and faces the high e_P^n, and the opportunistic inventor experiences a private challenge as well as the low e_P^c. The following proposition confirms this scenario.

Proposition 4. *(Pre-grant challenges and reverse case selection). Suppose that the challenger can only file a challenge at the pre-grant stage. There is a PBE where only the opportunistic inventor is challenged when*

$$\frac{(1-\bar{\theta}\bar{e}_C)(1-\bar{\theta}e_P^c)}{(1-\bar{\theta}e_P^n)} \geq \frac{\pi-\underline{s}}{\pi} \geq \frac{(1-\underline{\theta}\bar{e}_C)(1-\underline{\theta}e_P^c)}{(1-\underline{\theta}e_P^n)}, \tag{B.1}$$

where $\underline{s} = [u_C(\underline{\theta}) + (1-\underline{\theta}\underline{e}_C)\underline{\theta}e_P^c b]/(1-\underline{\theta}e_P^n)$.

Proof. In a separating equilibrium where only the true inventor settles, along the equilibrium path the settlement payment $\underline{s}$ is determined by the challenger's indifference between accepting the offer or litigating against the true inventor. Note that upon settlement, the challenger receives $\underline{s}$ only when the application survives subsequent public enforcement e_P^n. And the opportunistic inventor faces private challenge efforts $\bar{e}_C$, and public examination e_P^c if survives the challenge. Condition (B.1) comes from that neither type of inventor is willing to deviate to mimic the other type. That is, the true inventor prefers paying $\underline{s}$ than encountering two stages of enforcement, $(1-\underline{\theta}e_P^n)(\pi-\underline{s}) \geq (1-\underline{\theta}\bar{e}_C)(1-\underline{\theta}e_P^c)\pi$; and the opportunistic inventor prefers examination than settlement, $(1-\bar{\theta}\bar{e}_C)(1-\bar{\theta}e_P^c)\pi \geq (1-\bar{\theta}e_P^n)(\pi-\underline{s})$. To support this equilibrium, the challenger accepts any deviant offer $s' > \underline{s}$, and rejects any $s' < \underline{s}$ whiling litigating with efforts $\bar{e}_C$. *Q.E.D.*

First note that condition (B.1) won't hold when $e_P^c = 0$. For in this case, a necessary condition of this equilibrium,

$$\frac{(1-\bar{\theta}\bar{e}_C)(1-\bar{\theta}e_P^c)}{(1-\bar{\theta}e_P^n)} \geq \frac{(1-\underline{\theta}\bar{e}_C)(1-\underline{\theta}e_P^c)}{(1-\underline{\theta}e_P^n)}, \tag{B.2}$$

reduces to $e_P^n \geq \bar{e}_C$, contradictory with

$$\frac{1-\underline{\theta}\bar{e}_C}{1-\underline{\theta}e_P^n} \leq \frac{\pi-\underline{s}}{\pi} < 1. \tag{B.3}$$

In order to consider when it's more likely to have this equilibrium, let us fix $\bar{e}_C$, $\underline{\theta}$, and e_P^c at strictly positive levels, but less than one. Suppose that $\underline{s}$ is small enough (due to, say, a small b) so that

$$\frac{\pi-\underline{s}}{\pi} \geq (1-\underline{\theta}\bar{e}_C)\frac{1-\underline{\theta}e_P^c}{1-\underline{\theta}} \geq (1-\underline{\theta}\bar{e}_C)\frac{1-\underline{\theta}e_P^c}{1-\underline{\theta}e_P^n}. \tag{B.4}$$

That is, the second inequality in condition (B.1) holds for all e_P^n. In this case, the separating equilibrium exists as long as

$$(1-\bar{\theta}\bar{e}_C)\frac{1-\bar{\theta}e_P^c}{1-\bar{\theta}e_P^n} \geq 1 \Rightarrow \frac{1-\bar{\theta}e_P^c}{1-\bar{\theta}e_P^n} \geq \frac{1}{1-\bar{\theta}\bar{e}_C}. \tag{B.5}$$

For all possible $\bar{\theta}$, it is more likely to hold as e_P^n grows larger. In the extreme case of $\bar{\theta} = 1$, this condition is guaranteed when e_P^n is large enough. This equilibrium exists exactly when the weak patent is of the worst kind, and the patent office exerts maximal efforts to eliminate it when trying to exploit the information provided by case selection.

Remark. (Can sequential private challenges reverse the pattern?) One might suspect that the reverse pattern of case selection is generated by sequential efforts to eliminate patent applications, and could happen as well under post-grant challenges with multiple potential challengers.

For simplicity, suppose there are two potential challengers C_1 and C_2, with identical cost $c(\cdot)$ and benefit b . If the inventor's bargaining with C_1 results in the litigation of opportunistic inventor and settlement of true inventor, then C_1 exerts litigation efforts $\bar{e}_C$. Denote the true inventor's settlement offer as s_1. This separating equilibrium fully reveals the inventor's type, and so, knowing the litigation history, there will be no litigation between C_2 and the inventor (when the opportunistic inventor survives C_1's challenge). C_2 will settle with the good (opportunistic) inventor with a payment $u_C(\underline{\theta})$ ($u_C(\bar{\theta})$, respectively). Since

$$\pi - s_1 - u_C(\underline{\theta}) \geq (1-\underline{\theta}\bar{e}_C)\pi - u_C(\bar{\theta}) > (1-\bar{\theta}\bar{e}_C)\pi - u_C(\bar{\theta}), \tag{B.6}$$

the opportunistic inventor will deviate to mimic the true inventor. The reverse pattern of case selection will not happen under sequential private challenges. ■

Now, consider a potential challenger's timing choice. Suppose that both pre- and post-grant challenges are available to the challenger, but there is only one challenge opportunity. In the absence of a settlement agreement, with belief α and corresponding θ^0,[33] the challenger's payoff from initiating a pre-grant challenge is $u_C(\theta^0) + [1 - \theta^0 e_C^*(\theta^0)]e_P^c \theta^0 b$. If the challenger waits after the patent issuance, his expected payoff is $\theta^0 e_P^n b + (1-\theta^0 e_P^n)u_C(\hat{\theta})$, where $\hat{\theta} = \hat{\alpha}\underline{\theta} + (1-\hat{\alpha})\bar{\theta}$ and $\hat{\alpha}$ is determined according to condition (6), with $e_P = e_P^n$. Since $\hat{\alpha} > \alpha$ for all $e_P^n > 0$, $\hat{\theta} < \theta^0$, $e_C^*(\theta^0) > e_C^*(\hat{\theta})$, and $c(e_C^*(\theta^0)) > c(e_C^*(\hat{\theta}))$. We should expect more intensive private challenge efforts at the pre-grant stage than at the post-grant stage.

Because

$$\begin{aligned} & u_C(\theta^0) + [1-\theta^0 e_C^*(\theta^0)]e_P^c\theta^0 b \\ < \quad & \theta^0 e_C^*(\theta^0) b - (1-\theta^0 e_P^n)c(e_C^*(\theta^0)) + [1-\theta^0 e_C^*(\theta^0)]e_P^c\theta^0 b \\ = \quad & -(1-\theta^0 e_P^n)c(e_C^*(\theta^0)) + b\Big[\theta^0 e_C^*(\theta^0) + (1-\theta^0 e_C^*(\theta^0))\theta^0 e_P^c\Big], \end{aligned} \tag{B.7}$$

and

$$\begin{aligned} & \theta^0 e_P^n b + (1-\theta^0 e_P^n)u_C(\hat{\theta}) \\ = \quad & -(1-\theta^0 e_P^n)c(e_C^*(\hat{\theta})) + b\Big[\theta^0 e_P^n + (1-\theta^0 e_P^n)\hat{\theta} e_C^*(\hat{\theta})\Big], \end{aligned} \tag{B.8}$$

a sufficient condition for the challenger to choose the post-grant procedure is

$$e_P^n - e_P^c > e_C^*(\theta^0)(1-\theta^0 e_P^c). \tag{B.9}$$

This condition is more likely to hold as e_P^n gets larger and e_P^c gets smaller. That is, the challenger will postpone and free ride on public efforts if the patent office targets and exert much higher efforts towards those applications not being protested by private players.

Proposition 5. *(Timing to challenge). When condition (B.9) holds, a potential challenger prefers to challenge at the post-grant stage.*

C Alternative Settings (Not for publication)

This appendix extends our main results to settings where (i) the potential challenger makes the settlement offer; or (ii) the inventor's possible types are continuous.

[33] This α may be the initial belief when there is no bargaining at all between the inventor and challenger, or the belief after the breakdown of a settlement negotiation.

☐ **When the challenger makes the offer:** Assign the bargaining power to the challenger in the two-type case. Given belief $\hat{\alpha}$ (and so average invalidity $\hat{\theta}$), if the challenger decides not to settle at all, his expected litigation payoff is $u_C(\hat{\theta})$. If he wants to settle only with the opportunistic inventor, the settlement offer (the payoff he promises to the inventor) is $u_I(\bar{\theta}, \underline{e}_C)$, and he will exert effort $\underline{e}_C$ against the true inventor (recall that this effort cannot be part of the settlement agreement). His payoff under "partial settlement" is $\hat{\alpha} u_C(\underline{\theta}) + (1-\hat{\alpha})[\pi - u_I(\bar{\theta}, \underline{e}_C)]$.

To fully settle the case the inventor's willingness to accept the challenger's offer depends on the e_C at the off-path event of litigation, and a higher e_C pushes down the settlement offer. But next proposition shows that only $\underline{e}_C$ fulfills the criterion $D1$.[34] By offering $u_I(\underline{\theta}, \underline{e}_C)$, the challenger's payoff from fully settlement is $\pi - u_I(\underline{\theta}, \underline{e}_C)$. Define the following terms:

$$\bar{\alpha}_1 : \pi - u_I(\underline{\theta}, \underline{e}_C) \equiv \bar{\alpha}_1 u_C(\underline{\theta}) + (1-\bar{\alpha}_1)[\pi - u_I(\bar{\theta}, \underline{e}_C)], \tag{C.1}$$

$$\bar{\alpha}_2 : u_C(\bar{\alpha}_2 \underline{\theta} + (1-\bar{\alpha}_2)\bar{\theta}) \equiv \pi - u_I(\underline{\theta}, \underline{e}_C), \tag{C.2}$$

$$\bar{\alpha}_3 : u_C(\bar{\alpha}_3 \underline{\theta} + (1-\bar{\alpha}_3)\bar{\theta}) \equiv \bar{\alpha}_3 u_C(\underline{\theta}) + (1-\bar{\alpha}_3)[\pi - u_I(\bar{\theta}, \underline{e}_C)], \text{ s.t. } \bar{\alpha}_3 < 1. \tag{C.3}$$

$\bar{\alpha}_1$ is the cutoff level where the challenger is indifferent between full settlement and settling only with the opportunistic inventor (partial settlement). By the same token, $\bar{\alpha}_2$ is the cutoff where the challenger is indifferent between no settlement at all and full settlement; and $\bar{\alpha}_3$ the cutoff for indifference between no settlement and partial settlement. Note that $\bar{\alpha}_1 \in (0,1)$ is always well-defined, but there not may exist $\bar{\alpha}_2$ and $\bar{\alpha}_3$ in the open interval $(0,1)$.

Proposition 6. *(Bargaining equilibria when the challenger makes the offer). Let the challenger make the settlement offer. Suppose that the inventor agrees to settle whenever she is indifferent between settlement or not, the offer to fully settle the case in a PBE surviving D1 is $u_I(\underline{\theta}, \underline{e}_C)$. In this case, the weak patent is fully exposed to private enforcement only when $u_I(\bar{\theta}, \underline{e}_C) > \pi - u_C(\bar{\theta})$, and (i) $\hat{\alpha} < \bar{\alpha}_2$, in the case of $\bar{\alpha}_1 \leq \bar{\alpha}_2$; or (ii) $\hat{\alpha} < \bar{\alpha}_3$, in the case of $\bar{\alpha}_1 > \bar{\alpha}_2$. Otherwise, either there is no litigation or only the true inventor litigates.*

Suppose that the inventor may also respond to the challenger's offer in mixed strategies, then the challenger's payoff is strictly higher when the weak patent is only partially exposed to private enforcement than when full exposure. When $u_I(\bar{\theta}, \underline{e}_C) > \pi - u_C(\bar{\theta})$

[34] However, the general pattern of bargaining outcomes is not affected by this selection.

and $\hat{\alpha}$ small enough so that full litigation is optimal in the previous case, it is optimal for the challenger to make a settlement offer $u_I(\bar{\theta}, e_C^*(\theta_z))$ and exert litigation efforts $e_C^*(\theta_z)$ such that the opportunistic inventor will litigate with probability $z \in (0,1)$ and the true inventor will always litigate, where $\theta_z = \alpha_z \underline{\theta} + (1-\alpha_z)\bar{\theta}$ and $\alpha_z \equiv \hat{\alpha}/[\hat{\alpha} + (1-\hat{\alpha})z] \in (\hat{\alpha}, 1)$. The challenger's payoff is

$$\max_{\alpha_z} U_z = \frac{\hat{\alpha}}{\alpha_z} u_C(\theta_z) + (1 - \frac{\hat{\alpha}}{\alpha_z})[\pi - u_I(\bar{\theta}, e_C^*(\theta_z))]. \tag{C.4}$$

Proof. Suppose that the inventor will agree to settle upon indifference. To fully settle the case, the challenger needs to offer a payoff $u_I(\underline{\theta}, e)$, where $e \in [\underline{e}_C, \bar{e}_C]$ is determined by the challenger's off-path belief should the inventor reject the offer. The lowest offer, $u_I(\underline{\theta}, \bar{e}_C)$, is supported by the belief that the rejection must come from the opportunistic inventor. According to Lemma 2, however, this belief fails $D1$. The lemma also shows that the only off-path belief surviving $D1$ is that such rejection must be from the good type; and so the offer could be supported by a PBE with $D1$ is $u_I(\underline{\theta}, \underline{e}_C)$. By comparing the challenger's payoffs from different settlement policies, we get the range of $\hat{\alpha}$ such that the challenger will not settle at all.

Suppose that the inventor can respond to the challenger's offer with mixed strategies. First note that it won't be in the challenger's interests to induce mixed strategy responses from the true inventor. In that case, the challenger offers a payoff $u_I(\underline{\theta}, \underline{e}_C)$ so that the true inventor is indifferent between settlement and litigation; and since the opportunistic inventor always settles, the litigation effort is $\underline{e}_C$. The true inventor's probability of acceptance will only change the belief upon settlement, but neither the settlement offer nor the litigation effort. By $\pi - u_I(\underline{\theta}, \underline{e}_C) > u_C(\underline{\theta})$, the challenger's payoff is increasing in the probability of the true inventor's settlement; the challenger can increase his offer by a very small amount to guarantee full settlement.

Let the opportunistic inventor adopt mixed strategy responses. Given $\hat{\alpha}$, if she litigates with probability $z \in (0,1)$ upon indifference, then the challenger's belief upon litigation becomes $\alpha_z \equiv \hat{\alpha}/[\hat{\alpha} + (1-\hat{\alpha})z] \in (\hat{\alpha}, 1)$, and litigation efforts $e_C^*(\theta_z) \in (\underline{e}_C, e_C^*(\hat{\theta}))$. As z increases, α_z decreases and $e_C^*(\theta_z)$ increases. For the opportunistic inventor to be indifferent, the challenger offers a settlement payoff $u_I(\bar{\theta}, e_C^*(\theta_z))$. By doing so, the challenger's payoff is

$$
\begin{aligned}
U_z =& \hat{\alpha}[\underline{\theta} e_C^*(\theta_z) b - c(e_C^*(\theta_z))] \\
& + (1-\hat{\alpha})\Big\{ z[\bar{\theta} e_C^*(\theta_z) b - c(e_C^*(\theta_z))] + (1-z)[\pi - u_I(\bar{\theta}, e_C^*(\theta_z))] \Big\} \\
=& [\hat{\alpha} + (1-\hat{\alpha})z] u_C(\theta_z) + (1-\hat{\alpha})(1-z)[\pi - u_I(\bar{\theta}, e_C^*(\theta_z))] \\
=& \frac{\hat{\alpha}}{\alpha_z} u_C(\theta_z) + (1 - \frac{\hat{\alpha}}{\alpha_z})[\pi - u_I(\bar{\theta}, e_C^*(\theta_z))].
\end{aligned} \tag{C.5}
$$

The challenger can obtain a payoff $U_z(\alpha_z)$, with any $\alpha_z \in (\hat{\alpha}, 1)$, when opportunistic inventor sets $z = [\hat{\alpha}(1-\alpha_z)]/[(1-\hat{\alpha})\alpha_z]$.

Note that as $\alpha_z \to \hat{\alpha}$, $U_z \to u_C(\hat{\theta})$, the challenger's payoff under no settlement; and

$$
\begin{aligned}
\frac{du_C(\theta_z)}{d\alpha_z}\Big|_{\alpha_z=\hat{\alpha}} &= \frac{1}{\hat{\alpha}}[\pi - u_I(\bar{\theta}, e_C^*(\hat{\theta})) - u_C(\hat{\theta})] + \frac{du_C(\hat{\theta})}{d\hat{\alpha}} + (1 - \frac{\hat{\alpha}}{\hat{\alpha}})\frac{du_I(\bar{\theta}, e_C^*(\hat{\theta}))}{de_C}\frac{\partial e_C^*(\hat{\theta})}{\partial \hat{\alpha}} \\
&= \frac{1}{\hat{\alpha}}\Big[\pi - u_I(\bar{\theta}, e_C^*(\hat{\theta})) - u_C(\hat{\theta}) - (\bar{\theta} - \underline{\theta}) e_C^*(\hat{\theta}) b\Big] \\
&> \frac{1}{\hat{\alpha}}\bar{\theta}(\pi - b) e_C^*(\hat{\theta}).
\end{aligned} \tag{C.6}
$$

Full litigation is strictly dominated when the opportunistic inventor plays mixed strategies. This implies that, when $\hat{\alpha}$ is small enough so that the challenger doesn't want to settle at all in case where the inventor always settles upon indifference, it is optimal for the challenger to obtain a payoff U_z. On the other hand, when $\hat{\alpha} \to 1$, the feasible set of α_z, $(\hat{\alpha}, 1)$ shrinks, and $U_z \to u_C(\underline{\theta})$, which is strictly smaller than $\pi - u_I(\underline{\theta}, \underline{e}_C)$, the payoff from full litigation. Therefore for $\hat{\alpha}$ large enough, it won't be optimal for the challenger to induce mixed-strategy response from the inventor. *Q.E.D.*

Changing the distribution of bargaining power doesn't change the necessary condition for the weak patent to have a strictly positive litigation probability. However, since $u_C(\hat{\theta})$ is increasing in $\hat{\theta}$ and so decreasing in $\hat{\alpha}$, a higher patent quality makes settlement more attractive to the challenger. Unlike the case where the inventor makes the offer, in this case the opportunistic inventor is fully exposed to private enforcement only when the patent quality is low enough. This is the major difference between the two distributions of bargaining power.

But, in fact, in this case the full and partial exposure regimes take place for the same range of $\hat{\alpha}$. Different regimes ensue depending on whether the inventor is allowed to play mixed strategies, and the challenger's payoff improves when the opportunistic inventor can be induced to play mixed strategies in a proper manner, and so only litigates with some probability.

Consider the impact of e_P on different regimes. Under full exposure, there is no settlement, and the challenger's litigation effort is $e_C^*(\hat{\theta})$. The crowding out effect of public enforcement thus is robust to the distribution of bargaining power. The following proposition shows that the positive relationship between public and private enforcement under partial exposure still holds provides additional conditions are imposed.

Proposition 7. *(Partial exposure when the challenger makes the offer). When the challenger makes the offer, the weak patent may encounter a private challenge only when $u_I(\bar{\theta}, \underline{e}_C) > \pi - u_C(\bar{\theta})$, and at the full exposure regime a higher e_P reduces the challenger's litigation efforts.*

Under the partial exposure, if the challenger's cost function satisfies $c^{'''} \geq 0$ and $\hat{\alpha}$ is small enough, then the challenger's litigation efforts is independent of e_P and the opportunistic inventor's litigation probability is increasing in e_P.

Proof. When the challenger makes the offer and the opportunistic inventor litigates with probability $z \in (0,1)$ upon indifference, by the proof of Proposition 6 for $\hat{\alpha}$ smaller than $\bar{\alpha}_2$ or $\bar{\alpha}_3$, depending on $\bar{\alpha} \gtrless \bar{\alpha}_2$, it is optimal for the challenger to induce the mixed-strategy response from the opportunistic inventor and obtain a payoff U_z for some z.

Given such $\hat{\alpha}$, denote $\alpha_z^* \in (\hat{\alpha}, 1)$ as the optimal belief upon litigation (derived from the optimal z^*), and $\theta_z^* = \alpha_z^*\underline{\theta} + (1-\alpha_z^*)\bar{\theta}$. The challenger's optimal payoff is

$$\begin{aligned} U_z(\theta_z^*) &= \frac{\hat{\alpha}}{\alpha_z^*} u_C(\theta_z^*) + (1 - \frac{\hat{\alpha}}{\alpha_z^*})[\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*))] \\ &= \pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - \frac{\hat{\alpha}}{\alpha_z^*}\Big[\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - u_C(\theta_z^*)\Big]. \end{aligned} \tag{C.7}$$

When $c^{'''} \geq 0$, for all $\hat{\alpha}$, U_z is strictly convex in α_z: The first derivative is

$$\frac{\partial U_z}{\partial \alpha_z} = \bar{\theta}\pi \frac{\partial e_C^*(\theta_z)}{\partial \alpha_z} + \frac{\hat{\alpha}}{\alpha_z^2}[\bar{\theta}\pi e_C^*(\theta_z) - u_C(\theta_z)] - \frac{\hat{\alpha}}{\alpha_z}[\bar{\theta}\pi \frac{\partial e_C^*(\theta_z)}{\partial \alpha_z} + (\bar{\theta} - \underline{\theta}) b e_C^*(\theta_z)], \tag{C.8}$$

and the second derivative is

$$\begin{aligned} \frac{\partial^2 U_z}{\partial \alpha_z^2} = &- \frac{2\hat{\alpha}}{\alpha_z^3}\Big[\bar{\theta} e_C^*(\theta_z)(\pi - \alpha_z b) + c(e_C^*(\theta_z)) + (\bar{\theta} - \underline{\theta})\alpha_z b \frac{\bar{\theta}(\pi - \alpha_z b) + \underline{\theta}\alpha_z b}{c''(e_C^*(\theta_z))}\Big] \\ &+ \bar{\theta}\pi(1 - \frac{\hat{\alpha}}{\alpha_z})\frac{\partial^2 e_C^*(\theta_z)}{\partial \alpha_z^2} < 0, \end{aligned} \tag{C.9}$$

where

$$\frac{\partial^2 e_C^*(\theta_z)}{\partial \alpha_z^2} = \frac{c'''}{(c'')^2}(\bar{\theta} - \underline{\theta}) b \frac{\partial e_C^*(\theta_z)}{\partial \alpha_z} \leq 0. \tag{C.10}$$

Together with $\partial U_z / \partial \alpha_z > 0$ as $\alpha_z \to \hat{\alpha}$ and $U_z \to \hat{\alpha} u_C(\underline{\theta}) + (1 - \hat{\alpha})[\pi - u_I(\bar{\theta}, \underline{e}_C)]$ as $\alpha_z \to 1$, the generalized program $\max_{\alpha_z} U_z$ has a unique solution over $\alpha_z \in (\hat{\alpha}, 1]$. If $\partial U_z / \partial \alpha_z < 0$ as $\alpha_z \to 1$, then the optimal $\alpha_z^* \in (\hat{\alpha}, 1)$; and if $\partial U_z / \partial \alpha_z \geq 0$ as $\alpha_z \to 1$, then we have a corner solution and the challenger should fully settle with the opportunistic inventor. In the former case, as $\alpha_z \to 1$, the first-order condition,

$$\left. \frac{\partial U_z}{\partial \alpha_z} \right|_{\alpha_z \to 1} = \bar{\theta} \pi \left. \frac{\partial e_C^*(\theta_z)}{\partial \alpha_z} \right|_{\alpha_z \to 1} + \hat{\alpha} \left[\bar{\theta} \pi \underline{e}_C - u_C(\underline{\theta}) - \bar{\theta} \pi \left. \frac{\partial e_C^*(\theta_z)}{\partial \alpha_z} \right|_{\alpha_z \to 1} + (\bar{\theta} - \underline{\theta}) b e_C^*(\theta_z) \right], \tag{C.11}$$

becomes strictly negative for $\hat{\alpha}$ small enough, i.e., we must have an interior solution.

Suppose that $\hat{\alpha}$ is so small that the optimal $\alpha_z^* \in (\hat{\alpha}, 1)$. Considering a small increase in the patent quality $\hat{\alpha}' > \hat{\alpha}$, we show that the same α_z^* remains optimal when $\hat{\alpha}'$ is close enough to $\hat{\alpha}$. Let $\hat{\alpha}'$ be close enough to $\hat{\alpha}$ so that $\alpha_z^* \in (\hat{\alpha}', 1)$. We want to show that $\forall \alpha' \in (\hat{\alpha}', 1)$ and $\alpha' \neq \alpha_z^*$, with $\theta' = \alpha' \underline{\theta} + (1 - \alpha') \bar{\theta}$,

$$\begin{aligned} &\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - \frac{\hat{\alpha}'}{\alpha_z^*} \left[\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - u_C(\theta_z^*) \right] \\ >&\pi - u_I(\bar{\theta}, e_C^*(\theta')) - \frac{\hat{\alpha}'}{\alpha'} \left[\pi - u_I(\bar{\theta}, e_C^*(\theta')) - u_C(\theta') \right], \end{aligned} \tag{C.12}$$

$$\Rightarrow u_I(\bar{\theta}, e_C^*(\theta')) - u_I(\bar{\theta}, e_C^*(\theta_z^*)) > \hat{\alpha}' \left\{ \frac{\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - u_C(\theta_z^*)}{\alpha_z^*} - \frac{\pi - u_I(\bar{\theta}, e_C^*(\theta')) - u_C(\theta')}{\alpha'} \right\}. \tag{C.13}$$

By the definition and uniqueness of α_z^*, since α' is also available under $\hat{\alpha}$ (for $(\hat{\alpha}', 1) \subset (\hat{\alpha}, 1)$),

$$\begin{aligned} &\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - \frac{\hat{\alpha}}{\alpha_z^*} \left[\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - u_C(\theta_z^*) \right] \\ >&\pi - u_I(\bar{\theta}, e_C^*(\theta')) - \frac{\hat{\alpha}}{\alpha'} \left[\pi - u_I(\bar{\theta}, e_C^*(\theta')) - u_C(\theta') \right] \end{aligned} \tag{C.14}$$

$$\Rightarrow u_I(\bar{\theta}, e_C^*(\theta')) - u_I(\bar{\theta}, e_C^*(\theta_z^*)) > \hat{\alpha} \left\{ \frac{\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - u_C(\theta_z^*)}{\alpha_z^*} - \frac{\pi - u_I(\bar{\theta}, e_C^*(\theta')) - u_C(\theta')}{\alpha'} \right\}. \tag{C.15}$$

Therefore, if $\alpha' < \alpha_z^*$, then $e_C(\theta') > e_C(\theta_z^*)$ and so $u_I(\bar{\theta}, e_C^*(\theta')) < u_I(\bar{\theta}, e_C^*(\theta_z^*))$, any $\hat{\alpha}' > \hat{\alpha}$ will fulfill our objective. The same is true if $\alpha' > \alpha_z^*$ but

$$\frac{\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - u_C(\theta_z^*)}{\alpha_z^*} \leq \frac{\pi - u_I(\bar{\theta}, e_C^*(\theta')) - u_C(\theta')}{\alpha'}. \tag{C.16}$$

On the other hand, if $\alpha' > \alpha_z^*$ and

$$\frac{\pi - u_I(\bar{\theta}, e_C^*(\theta_z^*)) - u_C(\theta_z^*)}{\alpha_z^*} > \frac{\pi - u_I(\bar{\theta}, e_C^*(\theta')) - u_C(\theta')}{\alpha'}, \tag{C.17}$$

a $\hat{\alpha}'$ close enough to $\hat{\alpha}$ guarantees the optimality of α_z^* under $\hat{\alpha}'$. *Q.E.D.*

□ **Continuous types:** Let the inventor keep the bargaining power but have continuous types $\theta \in [0, 1]$. Let *ex ante* (before the examination process begins) CDF be $F(\cdot)$ and *pdf* be $f(\cdot)$, with $f(\theta) > 0$ for all $\theta \in [0, 1]$. Again denote $\theta^0 \equiv \int_0^1 \theta dF$ as the *ex ante* expectation value of θ. A higher θ^0 implies a lower quality.

When all types of inventors file patent applications, under the post-grant challenge system and patent office efforts e_P, the probability to eliminate the application is $\int_0^1 \theta e_P dF = \theta^0 e_P$. Upon issuance, the distribution of θ is updated to

$$\hat{F}(\theta) \equiv \frac{1}{1 - \theta^0 e_P} \int_0^\theta (1 - \theta' e_P) dF \quad \text{and} \quad \hat{f}(\theta) \equiv \frac{1 - \theta e_P}{1 - \theta^0 e_P} f(\theta); \tag{C.18}$$

and the post-issuance expectation is

$$\hat{\theta} \equiv \int_0^1 \theta d\hat{F} = \frac{\theta^0 - e_P E(\theta^2)}{1 - e_P \theta^0}. \tag{C.19}$$

Intuitively, stronger public enforcement reduces $\hat{\theta}$:

$$\frac{\partial \hat{\theta}}{\partial e_P} = \frac{(\theta^0)^2 - E(\theta^2)}{(1 - e_P \theta^0)^2} \leq 0, \tag{C.20}$$

by Jensen's inequality and the fact that x^2 is a convex function.

To facilitate the presentation, let us define the following terms: given $\tilde{\theta} \in (0, 1)$,

$$\hat{\theta}^+ \equiv E(\theta | \theta \geq \tilde{\theta}, e_P) = \frac{1}{1 - \hat{F}(\tilde{\theta})} \int_{\tilde{\theta}}^1 \theta d\hat{F}, \tag{C.21}$$

and

$$\theta^+ \equiv E(\theta | \theta \geq \tilde{\theta}, e_P = 0) = \frac{1}{1 - F(\tilde{\theta})} \int_{\tilde{\theta}}^1 \theta dF. \tag{C.22}$$

$\hat{\theta}^+$ is the post-issuance expectation, conditional on θ greater than a threshold $\tilde{\theta}$; and θ^+ is the conditional mean at the *ex ante* stage, or, equivalently, when $e_P = 0$. By the same token, we define $\hat{\theta}^-$ and θ^- as the conditional expectations when $\theta \leq \tilde{\theta}$:

$$\hat{\theta}^- \equiv E(\theta | \theta \leq \tilde{\theta}, e_P) = \frac{1}{\hat{F}(\tilde{\theta})} \int_0^{\tilde{\theta}} \theta d\hat{F}, \tag{C.23}$$

and

$$\theta^{-} \equiv E(\theta|\theta \leq \tilde{\theta}, e_P = 0) = \frac{1}{F(\tilde{\theta})} \int_0^{\tilde{\theta}} \theta dF. \tag{C.24}$$

Maintain the assumption that the challenger's litigation effort e_C cannot be part of the settlement agreement. Denote again $u_C(E(\theta|\mathcal{L}))$ as the challenger's expected payoff when challenging a patent with expected "case quality" $E(\theta|\mathcal{L})$. When bargaining breaks down, the optimal litigation effort e_C^* also depends on $E(\theta|\mathcal{L})$: the first-order condition $E(\theta|\mathcal{L})b \equiv c'(e_C^*)$. Given e_C^*, a patentee with of type θ has a expected payoff from litigation $(1-\theta e_C^*)\pi$. Since $\theta = 0$ is always one of the possible types, $f(0) > 0$, and cannot be eliminated by the patent office, under asymmetric information full settlement cannot be a bargaining outcome. As long as $Pr(\theta > 0) > 0$, the challenger will not accept an agreement under which the inventor keeps the whole monopoly profit π.

For simplicity, consider only pure strategies. The following proposition, in resemblance of Lemma 1, shows that a settled patent dispute involves weak patents, i.e., those with high values of θ.

Proposition 8. *(Case selection under continuous types). Suppose that both private players use pure strategies. Whether the inventor or the challenger makes the settlement offer, there exists* $\tilde{\theta} \in (0, 1]$ *such that a patent-holder litigates when having types* $\theta' < \tilde{\theta}$*, and settles when having types* $\theta'' > \tilde{\theta}$*.*

Proof. Since only pure strategies are allowed, there is only one equilibrium settlement payment s (from the inventor to the challenger). Without loss of generality, let $s = 0$ if no agreement is ever reached. A bargaining outcome consists of two elements: the equilibrium settlement offer s and the challenger's litigation effort e_C^* in case of bargaining breakdown. The inventor's payoffs from settlement and litigation are $\pi - s$ and $(1 - \theta e_C^*)\pi$, respectively. The cut-off rule follows from the fact that the former is constant while the latter is decreasing in θ. *Q.E.D.*

By this proposition, the challenger's equilibrium litigation effort is determined in accordance with the expectation $E(\theta|\mathcal{L}) = \hat{\theta}^{-}$. Let $\bar{\theta}_I$ be the equilibrium cutoffs. We first derive a sufficient condition under which partial settlement can be supported by PBEs; then consider the impact of a marginal change in e_P and the possibility of a positive relationship between public and private enforcement.

Proposition 9. (Bargaining equilibrium with continuous types). Consider continuous types and let the inventor make the settlement offer. If $u_C(1) < e_C^*(\hat{\theta})\pi$, there is no PBE where all types of inventor litigate.

Any $\bar{\theta}_I \in (0,1)$ is an equilibrium cutoff of a PBE if it satisfies

$$\bar{\theta}_I e_C^*(\hat{\theta}^-)\pi \geq u_C(\hat{\theta}^+) \equiv \max_{e_C} \hat{\theta}^+ e_C b - c(e_C). \tag{C.25}$$

A sufficient condition for the existence of an equilibrium cutoff $\bar{\theta}_I \in (0,1)$ is

$$e_C^*\left(\frac{\theta^0 - E(\theta^2)}{1-\theta^0}\right)\pi > u_C(1) = \bar{e}_C b - c(\bar{e}_C), \tag{C.26}$$

where $\bar{e}_C = e_C^*(1) \leq 1$ is the maximal possible litigation effort, and $E(\theta^2)$ is evaluated at the ex ante distribution.

Proof. Firs, consider full litigation as the equilibrium outcome. The equilibrium litigation effort is $e_C^*(\hat{\theta})$, and equilibrium payoff for a patent-holder with type θ is $[1 - \theta e_C^*(\hat{\theta})]\pi$, decreasing in θ. To support this equilibrium, the challenger should reject any positive settlement offer with appropriate off-path beliefs. However, since the challenger will always agree to settle when offered a payment $u_C(1)$ (or plus a small amount in order to break the tie), the patentee with types close to $\theta = 1$ will find it profitable to deviate and settle when $\pi - u_C(1) > [1 - e_C^*(\hat{\theta})]\pi$.

Now, suppose that $\bar{\theta}_I \in (0,1)$ is an equilibrium cutoff, i.e., all $\theta' < \bar{\theta}_I$ litigate while all $\theta'' > \bar{\theta}_I$ settle. Let $\hat{\theta}^-$ and $\hat{\theta}^+$ be the conditional means corresponding to $\bar{\theta}_I$.

The type $\bar{\theta}_I$ must be indifferent between litigation (with a payoff $[1 - \bar{\theta}_I e_C^*(\hat{\theta}^-)]\pi$) and settlement (with a payoff $\pi - s$), otherwise she and adjacent types will move toward the more profitable strategy and upset the equilibrium. The equilibrium settlement payment is $s = \bar{\theta}_I e_C^*(\hat{\theta}^-)\pi$. But this offer has to be no smaller than the challenger's expected payoff from litigating against $\hat{\theta}^+$ in order to accept the offer. Thus determines condition (C.25). This equilibrium can be supported by the challenger's off-path responses to accept any deviant offers greater than $\bar{\theta}_I e_C^*(\hat{\theta}^-)\pi$, and reject smaller deviant offers while litigate with efforts at least as strong as the equilibrium litigation level $e_C^*(\hat{\theta}^-)$.

For existence, note that as $\bar{\theta}_I \to 1$, $\hat{\theta}^- \to \hat{\theta}$ and $\hat{\theta}^+ \to 1$. The right-hand side of condition (C.25) is simply the challenger's maximal possible payoff from litigation: $\max_\theta u_C(\theta) = u_C(1) = \bar{e}_C b - c(\bar{e}_C)$. The left-hand side, as $\bar{\theta}_I \to 1$, approaches to $e_C^*(\hat{\theta})\pi$, where $\hat{\theta}$ is decreasing in e_P. To guarantee the existence for all e_P, condition (C.26) establishes the existence when $e_P \to 1$. *Q.E.D.*

Given an equilibrium cutoff $\bar{\theta}_I \in (0,1)$, the equilibrium settlement payment and litigation efforts are $\bar{\theta}_I e_C^*(\hat{\theta}^-)\pi$ and $e_C^*(\hat{\theta}^-)$, respectively.

Remark. (Equilibrium refinement). As in a typical signaling game, multiple equilibria may ensue.[35] The intuitive criterion has no bites here.[36] And, different from the two-type case, a more stringent criterion such as $D1$ will eliminate all the PBEs with positive probability of settlement. This is because, for all deviant offers $s' \neq s$, those types $\theta'' > \bar{\theta}_I$ will be eliminated under $D1$ by the type $\bar{\theta}_I$: With the same equilibrium payoff but lower probability to be invalidated for all $e_C > 0$, whenever a type θ'' weakly prefers to deviate and offer s', the type $\bar{\theta}_I$ must strictly prefer to do so. But this implies that the highest possible off-path belief is $\bar{\theta}_I$, which busts the equilibrium since the challenger has no reasonable off-path belief to reject a deviant offer s' between $u_C(\bar{\theta}_I)$ and $u_C(\hat{\theta}^+)$. ■

We now proceed to consider the impact of public enforcement e_P. By $\hat{\theta}$ decreasing in e_P, a higher e_P makes it easier to sustain an equilibrium with no settlement. This corresponds to the "full exposure" regime in the two-type case, and requires that the worst type $\theta = 1$ be willing to mix with all other types and fact an litigation effort $e_C^*(\hat{\theta})$ rather than offering $u_C(1)$ to guarantee settlement. This would happen when e_P is high and so $e_C^*(\hat{\theta})$ is low enough.

Now, consider the effect of a marginal change in e_P. An increasing in e_P changes the distribution function $\hat{F}$ at the private bargaining stage: $\forall \theta < 1$,

$$\frac{\partial \hat{F}(\theta)}{\partial e_P} = \frac{\theta^0 - E(\theta' | \theta' \leq \theta)}{(1-\theta^0 e_P)^2} F(\theta) > 0. \tag{C.28}$$

A higher public enforcement effort shifts the distribution toward low values of θ. Presumably, this change may simultaneously move the equilibrium cutoff $\bar{\theta}_I$ and effort

[35] Indeed, when $\pi >> b$ such that

$$\pi \left[e_C^*(\hat{\theta}^-) + \bar{\theta}_I \frac{\partial e_C^*}{\partial \theta}\bigg|_{\hat{\theta}^-} \frac{\partial \hat{\theta}^-}{\partial \bar{\theta}_I} \right] > b e_C^*(\hat{\theta}^+) \frac{\partial \hat{\theta}^+}{\partial \bar{\theta}_I}, \tag{C.27}$$

for any $\bar{\theta}_I$ satisfies condition (C.25), so does any $\theta > \bar{\theta}_I$.

[36] A PBE here can be supported by off-path strategies such that the challenger accepts any deviant payment s' higher than s, and rejects any smaller payment while exerting litigation efforts no smaller than e_C^*. Both responses can be justified by a belief that this offer comes from an inventor with an average type $\hat{\theta}^+$. Note that for $s' < s$, no type of inventor can be eliminated by the intuitive criterion: Relative to their equilibrium payoffs, the challenger's acceptance of s' is strictly preferred by those $\theta'' > \bar{\theta}$, and the rejection with a litigation effort higher than e_C^* is strictly preferred by $\theta' \leq \bar{\theta}_I$. For the same reason, when $s' > s$, the intuitive criterion won't be able to eliminate a type $\theta' \leq \bar{\theta}_I$. So even if some types $\theta'' > \bar{\theta}_I$ can be deleted, a belief that a deviant offer comes from those types smaller than $\bar{\theta}_I$, with the resulting average quality $\hat{\theta}^-$, suffices to support the challenger's response.

e_C^*, with the latter both affected by the distribution and the equilibrium cutoff. For simplicity, we restrict attention to a particular type of equilibrium adjustment. Similar to the partial exposure regime under the two-type case, we consider when an increase in e_P will raise $\bar{\theta}_I$ but keep e_C^* unchanged. If this holds, then a higher public effort enlarges the set of inventor types under private scrutiny without compromising challenge efforts.

We consider a pair of change de_P and $d\bar{\theta}_I$ that keeps $\hat{\theta}^-$ unchanged, and so the equilibrium effort e_C^* unchanged, and test when this pair of changes still satisfies condition (C.25). Formally, define $\Lambda \equiv \bar{\theta}_I e_C^* \pi - u_C(\hat{\theta}^+)$. In a PBE, $\Lambda \geq 0$. We consider $(de_P, d\bar{\theta}_I)$ such that

$$\frac{\partial \Lambda}{\partial e_P} de_P + \frac{\partial \Lambda}{\partial \bar{\theta}_I} d\bar{\theta}_I \geq 0 \quad s.t. \quad \frac{\partial \hat{\theta}^-}{\partial e_P} de_P + \frac{\partial \hat{\theta}^-}{\partial \bar{\theta}_I} d\bar{\theta}_I = 0. \tag{C.29}$$

Proposition 10. *(Public and private enforcement under continuous types). In the continuous-type setting where the inventor makes the offer, a higher e_P makes it more likely to have all types of inventor involved in litigation. Full exposure occurs under high public enforcement.*

In a PBE with equilibrium cutoff $\bar{\theta}_I \in (0,1)$, a pair $(de_P, d\bar{\theta}_I)$ satisfies condition (C.29) if

$$\frac{\partial \hat{\theta}^- / \partial e_P}{\partial \hat{\theta}^- / \partial \bar{\theta}_I} \geq \frac{\partial \hat{\theta}^+ / \partial e_P}{\partial \hat{\theta}^+ / \partial \bar{\theta}_I}. \tag{C.30}$$

Under ex ante uniform distribution $F(\theta) = \theta$, condition (C.30) is satisfied when $\bar{\theta}_I$ is small enough.

Proof. Since $\hat{\theta}^-$ and so the equilibrium litigation effort e_C^* are not affected by the changes of e_P and $\bar{\theta}_I$, and by definition, $u_C(\hat{\theta}^+) = \hat{\theta}^+ e_C^*(\hat{\theta}^+) b - c(e_C^*(\hat{\theta}^+))$, we have

$$\frac{\partial \Lambda}{\partial e_P} = -e_C^*(\hat{\theta}^+) b \frac{\partial \hat{\theta}^+}{\partial e_P} \quad \text{and} \quad \frac{\partial \Lambda}{\partial \bar{\theta}_I} = e_C^*(\hat{\theta}^-)\pi - e_C^*(\hat{\theta}^+) b \frac{\partial \hat{\theta}^+}{\partial \bar{\theta}_I}. \tag{C.31}$$

By inserting the condition that keeps $\hat{\theta}^-$ intact,

$$d\bar{\theta}_I = -\frac{\partial \hat{\theta}^- / \partial e_P}{\partial \hat{\theta}^- / \partial \bar{\theta}_I} de_P, \tag{C.32}$$

and after a few algebraic manipulations, we get

$$\frac{\partial \Lambda}{\partial e_P} de_P + \frac{\partial \Lambda}{\partial \bar{\theta}_I} d\bar{\theta}_I = \frac{de_P}{\partial \hat{\theta}^- / \partial \bar{\theta}_I} \Big[-e_C^*(\hat{\theta}^-)\pi \frac{\partial \hat{\theta}^-}{\partial e_P} + e_C^*(\hat{\theta}^+) b \big(\frac{\partial \hat{\theta}^+}{\partial \bar{\theta}_I} \frac{\partial \hat{\theta}^-}{\partial e_P} - \frac{\partial \hat{\theta}^+}{\partial e_P} \frac{\partial \hat{\theta}^-}{\partial \bar{\theta}_I} \big) \Big]. \tag{C.33}$$

Since $\frac{\partial\hat{\theta}^-}{\partial\bar{\theta}_I} > 0 > \frac{\partial\hat{\theta}^-}{\partial e_P}$ (and so $d\bar{\theta}_I$ and de_P should have the same sign), the whole term is guaranteed to be positive if

$$\frac{\partial\hat{\theta}^+}{\partial\bar{\theta}_I}\frac{\partial\hat{\theta}^-}{\partial e_P} - \frac{\partial\hat{\theta}^+}{\partial e_P}\frac{\partial\hat{\theta}^-}{\partial\bar{\theta}_I} \geq 0, \tag{C.34}$$

or, equivalently, if condition (C.30) holds.

With *ex ante* uniform distribution, $F(\theta) = \theta$, post-issuance CDF and pdf are, respectively,

$$\hat{F}(\theta) = \frac{1}{1-\theta^0 e_P}\int_0^\theta (1-\theta' e_P)d\theta' = \frac{\theta(2-\theta e_P)}{2-e_P} \quad \text{and} \quad \hat{f}(\theta) = \frac{2-2\theta e_P}{2-e_P}. \tag{C.35}$$

Given a cutoff $\bar{\theta}_I$, the conditional expectations are

$$\hat{\theta}^+ = \frac{2}{2-(1+\bar{\theta}_I)e_P}\left[\frac{1}{2}(1+\bar{\theta}_I) - \frac{e_P}{3}(1+\bar{\theta}_I+\bar{\theta}_I^2)\right] \quad \text{and} \quad \hat{\theta}^- = \frac{2\bar{\theta}_I}{2-\bar{\theta}_I e_P}\left(\frac{1}{2} - \frac{e_P}{3}\bar{\theta}_I\right). \tag{C.36}$$

Therefore,

$$\frac{\partial\hat{\theta}^+}{\partial\bar{\theta}_I} = \frac{2(1-\bar{\theta}_I e_P)[2(1-e_P)+(1-\bar{\theta}_I e_P)]}{3[2-(1+\bar{\theta}_I)e_P]^2}, \tag{C.37}$$

$$\frac{\partial\hat{\theta}^+}{\partial e_P} = -\frac{(1-\bar{\theta}_I)^2}{3[2-(1+\bar{\theta}_I)e_P]^2}, \tag{C.38}$$

$$\frac{\partial\hat{\theta}^-}{\partial e_P} = -\frac{\bar{\theta}_I^2}{3(2-\bar{\theta}_I e_P)^2}, \tag{C.39}$$

$$\frac{\partial\hat{\theta}^-}{\partial\bar{\theta}_I} = \frac{2(3-\bar{\theta}_I e_P)(1-\bar{\theta}_I e_P)}{3(2-\bar{\theta}_I e_P)^2}, \tag{C.40}$$

and condition (C.30) requires:

$$\frac{\partial\hat{\theta}^-/\partial e_P}{\partial\hat{\theta}^-/\partial\bar{\theta}_I} = -\frac{\bar{\theta}_I^2}{2(3-\bar{\theta}_I e_P)(1-\bar{\theta}_I e_P)} \geq \frac{\partial\hat{\theta}^+/\partial e_P}{\partial\hat{\theta}^+/\partial\bar{\theta}_I} = -\frac{(1-\bar{\theta}_I)^2}{2(1-\bar{\theta}_I e_P)[2(1-e_P)+(1-\bar{\theta}_I e_P)]}, \tag{C.41}$$

$$\Rightarrow \left(\frac{1-\bar{\theta}_I}{\bar{\theta}_I}\right)^2 \geq \frac{3-\bar{\theta}_I e_P - 2e_P}{3-\bar{\theta}_I e_P}. \tag{C.42}$$

$\bar{\theta}_I$ has to be small enough. For instance, it is satisfied for all $\bar{\theta}_I \leq \frac{1}{2}$. *Q.E.D.*

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